# ON DIFFERENTIAL EQUATION OF INVARIANTS OF BINARY FORM

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Abstract. An explicit form of single first order PDE for invariants of binary form are found. By solving the equation a minimal generation set for a ring of invariants and theirs syzygies are calculated in the cases $n \leq 6$ and $n = 8$.

## 1. Introduction

Let $V_n$ be a vector $k$-space of the binary forms of degree $n$

$$u(x, y) = \sum_{i=0}^{n} \alpha_i \binom{n}{i} x^{n-i} y^i,$$

where $\alpha_i \in k$, and $k$ is a field of characteristic zero. Let us indentify the coordinate ring $R_n$ of the space $V_n$ with the polinomial ring $k[\alpha_0, \alpha_1, \ldots, \alpha_n]$. The group $SL_2$ acts on $V_n$ by the rule

$$(g\, u)(x, y) = u(d\, x - b\, y, -c\, x + a\, y), \quad g = \begin{pmatrix} a\, b \\ c\, d \end{pmatrix} \in SL_2.$$

The generating elements $\begin{pmatrix} 0\,1 \\ 0\,0 \end{pmatrix}$, $\begin{pmatrix} 0\,1 \\ 0\,0 \end{pmatrix}$ of the tangent Lie algebra $\mathfrak{sl}_2$ act on $V_d$ by derivations $-y\frac{\partial}{\partial x}$, $-x\frac{\partial}{\partial y}$, see [1], and on $R_n$ by derivations

$$d_1 := \alpha_0 \frac{\partial}{\partial \alpha_1} + 2\, \alpha_1 \frac{\partial}{\partial \alpha_2} + \cdots + n\, \alpha_{n-1} \frac{\partial}{\partial \alpha_n},$$
$$d_2 := n\, \alpha_1 \frac{\partial}{\partial \alpha_0} + (n-1)\, \alpha_2 \frac{\partial}{\partial \alpha_1} + \cdots + \alpha_n \frac{\partial}{\partial \alpha_{n-1}}.$$

It follows that the invariant ring $R_n^{SL_2}$ coincides with a ring of polynomial solutions of the following first order PDE system, see [2], [3]:

$$\left\{ \begin{array}{r} \alpha_0 \dfrac{\partial u}{\partial \alpha_1} + 2\, \alpha_1 \dfrac{\partial u}{\partial \alpha_2} + \cdots + n\, \alpha_{n-1} \dfrac{\partial u}{\partial \alpha_n} = 0, \\ n\, \alpha_1 \dfrac{\partial u}{\partial \alpha_0} + (n-1)\, \alpha_2 \dfrac{\partial u}{\partial \alpha_1} + \cdots + \alpha_n \dfrac{\partial u}{\partial \alpha_{n-1}} = 0, \end{array} \right. , \qquad (*)$$

i.e. $R_n^{SL_2} = k[\alpha_0, \alpha_1, \ldots, \alpha_n]^{d_1} \bigcap k[\alpha_0, \alpha_1, \ldots, \alpha_n]^{d_2}$, where $u \in k[\alpha_0, \alpha_1, \ldots, \alpha_n]$, and

$$k[\alpha_0, \alpha_1, \ldots, \alpha_n]^{d_i} := \{ f \in k[\alpha_0, \alpha_1, \ldots, \alpha_n] | d_i(f) = 0 \}, i = 1, 2.$$

The ring of invariants and theirs syzygies was a major object of research in classical invariant theory of the 19th century. For $n \leq 6$ the invariant ring was described by Gordan [4]. The case $n$=8 was considered by Shioda [5]. Allmost all known invariants were found in implicit way by the so-called symbolic method. In this case every invariant is represented as an action of an invariant differential operator applied to covariants ($\Omega$ process). For $n \leq 5$ Faá de Bruno [8] and Sylvester [9] have calculated an explicit way a minimal generating set of the $R_n^{SL_2}$. The explicit form is highly unwieldy. For example, one invariant of the generating set for the ring $R_5^{SL_2}$ is a polynomial of degree 18, consisting of 848 terms.

For $n=5$, Sylvester also found the explicit form of single syzygy of degree 36 between the four invariant of the generating set. For $n=6$, there exists a unigue syzygy and in the case $n=8$ the 9 fundamental invariants are related by 5 syzygies, see [10], [5].
It is the aim of this paper to reduce the system $(*)$ to one equation and try to use it for calculation of invariants for small $n$. For each equations of the system $(*)$ one may easily find a fundamental system of solutions in the qoutiend field $k(\alpha_0,\alpha_1,\ldots,\alpha_n)$. Having the fundamental system for one derivation we may discover an action of another derivation on the fundamental system . In this way we succeed to reduce the system to single equation. By using the equation together with an additional information about invariants such as theirs degree and number of generators one may calculate an explicit form for invariants and their syzygies. In this paper we do so for $n\leq 6$ and $n=8$.

## 2. Differential eqation of invariants

From now, let us change the variable set $\alpha_0,\alpha_1,\ldots,\alpha_n$ on $x_0,x_1,\ldots,x_n$. Denote by $k[X]$ the ring $k[x_0,x_1,\ldots,x_n]$, and by $k(X)$ denote a quotient field of the ring $k[X]$. The derivations $d_1,d_2$ one may extend from $k[X]$ to $k(X)$ preserving the same notations $d_1,d_2$. It is evident that $k[X]^{d_i}=k(X)^{d_i}\bigcap k[X]$. The derivations $d_1$ is localy nilpotent on $k[X]$, moreover $d_1(\lambda)=-1$, where $\lambda=-\frac{x_1}{x_0}$. Therefore (see for example [6], Proposition 1.3.21) for the derivation $d_1$ we can get a description of the ring $k(X)^{d_1}$, namely

$$k(X)^{d_1}=k(\sigma(x_0),\sigma(x_1),\ldots,\sigma(x_n)),$$

where $\sigma:k[X]\to k[X]^{d_1}$ is a ring homomorfism defined by

$$\sigma(a)=\sum_{i=0}^{\infty}d_1^i(a)\frac{\lambda^i}{i!}.$$

It is well known ([11], Proposition 2.1) that in this case $k(X)=k(X)^{d_1}[\lambda]$ and $\lambda$ are algebraically independent over $k(X)^{d_1}$, therefore $k(X)$ is a polynomial ring in one variable over $k(\sigma(x_0),\sigma(x_2),\ldots,\sigma(x_n))$. This fact allows us to define an action of the derivation $d_2$ on $k(X)$ in new coordinates $\sigma(x_0),\sigma(x_2),\ldots,\sigma(x_n),\lambda$. Denote by $d$ the derivation

$$d:=d_2(\lambda)\,\frac{\partial}{\partial\lambda}+d_2(\sigma(x_0))\,\frac{\partial}{\partial\sigma(x_0)}+\ldots+d_2(\sigma(x_n))\,\frac{\partial}{\partial\sigma(x_n)}.$$

Since $k(X)=k(\sigma(x_0),\sigma(x_2),\ldots,\sigma(x_n))[\lambda]$ we are always able to express $d_2(\lambda)$, $d_2(\sigma(x_i))$ in terms of $\lambda,\sigma(x_0),\sigma(x_2),\ldots,\sigma(x_n)$, so the derivation $d$ is well defined.

It is clear that the ring $k[X]^d$ coincides with a ring polynomial solution of the following differential equation

$$d_2(\lambda)\,\frac{\partial u}{\partial\lambda}+d_2(\sigma(x_0))\,\frac{\partial u}{\partial\sigma(x_0)}+\ldots+d_2(\sigma(x_n))\,\frac{\partial u}{\partial\sigma(x_n)}=0, u\in k[\sigma(x_0),\sigma(x_1),\ldots,\sigma(x_n)], \quad (**)$$

and we get $R_n^{SL_2}=k[X]^d\bigcap k[X]$.

For example, let us consider the case $n=2$. The derivations $d_1$ and $d_2$ has form

$$d_1=x_0\,\frac{\partial}{\partial x_1}+2\,x_1\frac{\partial}{\partial x_2},$$
$$d_2=2\,x_1\,\frac{\partial}{\partial x_0}+x_2\frac{\partial}{\partial x_1}$$

Since

$$\begin{aligned}
&\sigma(x_0) = x_0,\\
&\sigma(x_1) = x_1 + x_0\,\lambda = x_1 + x_0\,(-\frac{x_1}{x_0}) = 0,\\
&u_2 := \sigma(x_2) = x_2 + 2\,x_1\,\lambda + 2\,x_0\,\frac{\lambda^2}{2!} = x_2 - x_0\,\lambda^2,
\end{aligned}$$

then $k(X)^{d_1} = k(x_0, u_2)$. Taking into account $x_2 = u_2 + x_0\,\lambda^2$ we obtain

$$\begin{aligned}
&d_2(x_0) = 2\,x_1 = -2x_0\,\lambda\\
&d_2(\lambda) = d_2(-\frac{x_1}{x_0}) = \lambda^2 - \frac{u_2}{x_0},\\
&d_2(u_2) := d_2\Big(x_2 - x_0\,\lambda^2\Big) = 2\,\lambda u_2,
\end{aligned}$$

Therefore, the derivation $d$ has form

$$d = -2x_0\,\lambda\frac{\partial}{\partial x_0} + \left(\lambda^2 - \frac{u_2}{x_0}\right)\frac{\partial}{\partial\lambda} + 2\,x_0 u_2\frac{\partial}{\partial u_2},$$

and the corresponding differential equation $(**)$ after multiplying by $x_0$ turns into

$$-2x_0^2\,\lambda\frac{\partial u}{\partial x_0} + (x_0\lambda^2 - u_2)\frac{\partial u}{\partial\lambda} + 2\,x_0^2 u_2\frac{\partial u}{\partial u_2} = 0.$$

It is well known that a first order PDE in three variables has a fundamental system which consists of no more than 2 solutions. For our equation it can easily be checked that the fundamental system, it is $x_0\,u_2$ and $u_2+x_0\lambda^2$, hence $k(x_0, u_2)[\lambda]^d = k(x_0\,u_2, u_2+x_0\lambda^2)$. Further, the ring $k(x_0\,u_2)$ obviously is the intersection of rings $k(x_0\,u_2, u_2 + x_0\lambda^2)$ and $k(x_0, u_2)$. Since $x_0\,u_2 = x_0\,x_2 - x_1^2$ already belongs to $k[X]$ it follows that the invariant ring $R_2^{SL_2}$ is generated by single invariant $x_0\,x_2 - x_1^2$.

Consider the case of arbitrary $n$. We have

$$k(X)^{d_1} = k(x_0, u_2, u_3, \ldots, u_n),$$

where

$$u_i := \sigma(x_i) = \sum_{k=0}^{i}\binom{i}{k}\,x_{i-k}\,\lambda^k.$$

First of all we will express variables $\lambda, x_2, \ldots, x_n$ through $u_2, \ldots, u_n$. Denote by $B_i$ the sum $ix_1\lambda^{i-1} + x_0\lambda^i = -(i-1)x_0\lambda^i$ of the last two terms of $u_i$. In particulary we obtain

$$\begin{aligned}
&u_2 = x_2 + B_2,\\
&u_3 = x_3 + 3\,x_2\lambda + B_3,\\
&u_4 = x_4 + 4\,x_3\lambda + 6\,x_2\lambda^2 + B_4,
\end{aligned}$$

Since

$$\begin{aligned}
&x_2 = u_2 - B_2,\\
&x_3 = u_3 - 3\,u_2\lambda - (B_3 - 3\,B_2\lambda),\\
&x_4 = u_4 - 4u_3\lambda + 6u_2\lambda^2 - (B_4 - 4B_3\lambda + 6B_2\lambda^2),
\end{aligned}$$

and for arbitrary $i$ :

$$x_i = \sum_{k=0}^{i-2}(-1)^k\binom{i}{k}u_{i-k}\lambda^k - \Big(\sum_{k=0}^{i-2}(-1)^k\binom{i}{k}B_{i-k}\lambda^k\Big).$$

Taking into account

$$\sum_{k=0}^{i-2}(-1)^k\binom{i}{k}B_{i-k}\lambda^k=\sum_{k=0}^{i-2}(-1)^{k+1}(i-k-1)\binom{i}{k}x_0\lambda^i=$$

$$=x_0\lambda^i\sum_{k=0}^{i-2}(-1)^{k+1}(i-k-1)\binom{i}{k}=(-1)^{i+1}x_0\lambda^i,$$

we get required form for $x_i$ :

$$x_i=\sum_{k=0}^{i-2}(-1)^k\binom{i}{k}u_{i-k}\lambda^k+(-1)^ix_0\lambda^i.$$

To wtite the equation $(**)$ we need to find an explicit expression of $d_2(u_i)$ in terms of $\lambda, x_2, \ldots, x_n$ through $u_2, \ldots, u_n$. By direct calculation we obtain

$$\begin{aligned}
&d_2(\lambda)=\lambda^2-(n-1)\frac{u_2}{x_0},\\
&d_2(u_2)=(n-2)u_3-(n-4)u_2\lambda,\\
&d_2(u_3)=(n-3)u_4-(n-6)u_3\lambda-3(n-1)\frac{u_2^2}{x_0}.
\end{aligned}$$

In the general case, for $u_i, i>3$ we have:

$$d_2(u_i)=\sum_{k=0}^{i-2}\binom{i}{k}d_2(x_{i-k})\,\lambda^k+\left(\sum_{k=1}^{i-2}kx_{i-k}\binom{i}{k}\lambda^{k-1}\right)d_2(\lambda)+d_2(B_i).$$

Let us calculate each sum separately

$$\begin{aligned}
&\sum_{k=0}^{i-2}\binom{i}{k}d_2(x_{i-k})\,\lambda^k=\sum_{k=0}^{i-2}\binom{i}{k}(n-(i-k))\,x_{i-k+1}\lambda^k=\\
&=\sum_{k=0}^{i-2}\binom{i}{k}(n-(i-k))\,\lambda^k\left(\sum_{s=0}^{i-k-1}(-1)^s\binom{i-k+1}{s}u_{i-k-s+1}\lambda^s+(-1)^{i-k+1}x_0\lambda^{i-k+1}\right)=\\
&=\sum_{k=0}^{i-2}\sum_{s=0}^{i-k-1}(-1)^s(n-(i-k))\binom{i}{k}\binom{i-k+1}{s}u_{i-k-s+1}\lambda^{s+k}+x_0\lambda^{i+1}T_i=\\
&=\sum_{p=2}^{i+1}u_p\lambda^{i+1-p}\sum_{s+k=i+1-p}(-1)^s(n-(i-k))\binom{i}{k}\binom{i-k+1}{s}+x_0\lambda^{i+1}T_i=\\
&=u_2\lambda^{i-1}S_2+\sum_{p=3}^{i+1}u_p\lambda^{i-p+1}\,S_p+x_0\lambda^{i+1}T_i=
\end{aligned}$$

here

$$\begin{aligned}
&T_i:=\sum_{k=0}^{i-2}(-1)^{i-k+1}(n-(i-k))\binom{i}{k},\\
&S_2:=\sum_{k=3}^{i+1}(-1)^{k-2}(n-(k-1))\binom{k}{2}\binom{i}{k-1},\\
&S_p:=\sum_{k=p}^{i+1}(-1)^{k-p}(n-(k-1))\binom{k}{p}\binom{i}{k-1}, p>2
\end{aligned}$$

**Lemma 1.** *The following equalities are hold:*

$$T_i = n + i - n\,i, i > 1.$$

*and for* $i > 3$

$$S_2 = -(n-1)i$$

$$S_p = \begin{cases} n-i, \text{for } p = i+1, \\ 2\,i - n, \text{for } p = i, \\ -i, \text{for } p = i-1, \\ 0, \text{for } 2 < p < i-1, \\ -(n-1)i, \text{for } p = 2. \end{cases}$$

*Proof.* Using the binomial identity

$$\sum_{k=0}^{i} (-1)^k \binom{i}{k} = \sum_{k=1}^{i-1} (-1)^{k-1} \binom{i-1}{k-1} = 0,$$

we get

$$T_i - n + (n-1)i = \sum_{k=0}^{i} (-1)^{i-k+1} (n-(i-k)) \binom{i}{k} = (n-i)(-1)^{i+1} \sum_{k=0}^{i} (-1)^{-k} \binom{i}{k} +$$

$$+(-1)^{i+1} \sum_{k=0}^{i} (-1)^{-k} k \binom{i}{k} = 0 + (-1)^{i+1} \sum_{k=1}^{i-1} (-1)^{k-1} \binom{i-1}{k-1} = 0.$$

The equalities for $S_p$ are derived from the following ortogonal relation ([7] )

$$\delta_{m,n} = \sum_{k=m}^{n} (-1)^k \binom{k}{m} \binom{n}{k}$$

in the same way. □

Hence we reduced the first sum to the form

$$(n-i)u_{i+1} - (n-2i)u_i\lambda - iu_{i-1}\lambda^2 - i(n-1)u_2\lambda^{i-1} + (n+i-n\,i)x_0\lambda^{i+1}.$$

Let us calculate the second sum in the expression of $d_2(u_i)$ :

$$\sum_{k=1}^{i-2} x_{i-k} k \binom{i}{k} \lambda^{k-1} d_2(\lambda) = \sum_{k=1}^{i-2} x_{i-k} i \binom{i-1}{k-1} \lambda^{k-1} d_2(\lambda) = i(u_{i-1} - B_{i-1}) d_2(\lambda).$$

and $d_2(B_i)$ :

$$d_2(B_i) = d_2(-(i-1)x_0\lambda^2) = (i-1)\lambda^{i-1}((n-i)x_0\lambda^2 + i(n-1)u_2).$$

Thus after all simplifications for $i > 3$ we get

$$d_2(u_i) = (n-i)u_{i+1} - (n-2i)u_i\lambda - i(n-1)\frac{u_2 u_{i-1}}{x_0}.$$

Consequently the derivation $d$ acts on $k(x_0, \lambda, u_2, \ldots, u_n)$ by the rule

$$\begin{aligned} &d(x_0) = -nx_0\lambda, \\ &d(\lambda) = \lambda^2 - (n-1)\frac{u_2}{x_0}, \\ &d(u_2) = (n-2)u_3 - (n-4)u_2\lambda, \\ &d(u_i) = (n-i)u_{i+1} - (n-2i)u_i\lambda - i(n-1)\frac{u_2 u_{i-1}}{x_0}, \text{for } i > 2. \end{aligned}$$

Finally, we obtain

**Theorem 1.** *The invariant ring $R_n^{SL_2}$ coincides with a ring of polynomial solutions of the following first order PDE*

$$-nx_0^2\lambda\frac{\partial u}{\partial x_0}+(x_0\lambda^2-(n-1)u_2)\frac{\partial u}{\partial \lambda}+((n-2)u_3x_0-(n-4)u_2x_0\lambda)\frac{\partial u}{\partial u_2}+$$
$$+\sum_{i=4}^{n}((n-i)u_{i+1}x_0-(n-2i)u_ix_0\lambda-i(n-1)u_2u_{i-1})\frac{\partial u}{\partial u_i}=0,$$

*where,*

$$\lambda=\frac{x_1}{x_0},$$
$$u_i=\sum_{k=0}^{i}\binom{i}{k}x_{i-k}\,\lambda^k,$$

*and $u\in k[X]\bigcap k[x_0,u_2,\dots,u_n]$.*

## 3. Solving of the invariant equation

Let us introduce on $k[x_0,u_2,\dots,u_n]$ three additional derivations $\hat{d}_1$,$\hat{d}_2$ and $e$ as follows

$$\begin{aligned}&\hat{d}_1(u_i)=i\,u_{i-1},\hat{d}_1(u_2)=0,\hat{d}_1(x_0)=0,\\&\hat{d}_2(u_i)=(n-i)\,u_{i+1},\hat{d}_2(x_0)=0,\\&e(u_i):=[\hat{d}_1,\hat{d}_2](u_i)=(n-2\,i)u_i,e(x_0)=n\,x_0.\end{aligned}$$

Then, in terms of these derivations, one may rewrite the derivation $d$ in the form

$$d=x_0\,\hat{d}_2-x_0\lambda e-(n-1)u_2\hat{d}_1. \qquad (***)$$

For the derivation $e$, every monomial $x_0^{\alpha_1}\,u_2^{\alpha_2}\,\cdots u_n^{\alpha_n}$ is an eigenvector with the eigenvalue

$$\omega(x_0^{\alpha_0}\,u_2^{\alpha_2}\,\cdots u_n^{\alpha_n})=n\,(\sum_i\,\alpha_i)-2\,(\alpha_2+2\,\alpha_2+\cdots+n\,\alpha_n).$$

A homogeneous polynomial is called isobaric if the sum $(2\,\alpha_2+3\,\alpha_3+\cdots+n\,\alpha_n)$ has an equal value on all monomials of the polynomial. The value is called $u$-weight of the polynomial $z$ and it is denoted by $\omega_u(z)$. Hence, if $z$ is isobaric polynomial then we have $e(z)=(n\deg(z)-2\,\omega_u(z))\,z$. Put

$$I_n:=\{z\in k[x_0,u_2,\dots,u_n],\ n\,\deg(z)-2\,\omega_u(z)=0\}.$$

It is clear that on isobaric polynomials the functions $\deg,\omega_u:k[x_0,u_2,\dots,u_n]\to\mathbb{Z}$ are additive. Thus $I_n$ form a subring of the ring $k[x_0,u_2,\dots,u_n]$. The following theorem describes solutions of the equation for the invariants.

**Theorem 2.** $R_n^{SL_2}=I_n\cap k[X]$.

*Proof.* Suppose that a polynomial $u\in R_n^{SL_2}$. Then $u\in k[x_0,u_2,\dots,u_n]$ and it is follow $\hat{d}_i(u)\in k[x_0,u_2,\dots,u_n], i=1,2$. Therefore the equality

$$d(u)=x_0\,\hat{d}_2(u)-x_0\lambda e(u)-(n-1)u_2\hat{d}_1(u)=0,$$

is possible only in the case if a coefficient by $\lambda$ is equal to zero, i.e. $e(u)=0$. Therefore $n\,\deg(u)-\omega_u(u)=0$ and $u\in I_n\cap k[X]$. So we get $R_n^{SL_2}\subset I_n\cap k[X]$.

Now suppose $u\in I_n\cap k[X]$. Let us show that $d_2(u)=0$. Define a $k$-linear, multiplicative map $\varphi:k[X]\to k[x_0,u_2,\dots,u_n]$ by the rule $\varphi(x_0)=x_0$, $\varphi(x_1)=0$ and $\varphi(x_i)=u_i$ for $2\le i\le n$.

**Lemma 2.** *If $z\in k[x_0,u_2,\dots,u_n]$, then $\varphi(z)=z$.*

*Proof.* Using the expansion

$$x_i = \sum_{k=0}^{i-2} (-1)^k \binom{i}{k} u_{i-k}\lambda^k + (-1)^i t\lambda^i,$$

we may every $x_i$ expres in form $x_i = u_i + \lambda\, u_i' = \varphi(x_i) + \lambda\, u_i'$ for some $u_i'$. Multiplicativity of maps $\varphi$ and $x_1 = -\lambda x_0 = \varphi(x_1) - \lambda x_0$, implies the existence of the representation $z = \varphi(z) + \lambda z'$ for arbitrary polynomial $z$ and for some polynomial $z'$. Since $z \in k[x_0, u_2, \ldots, u_n]$ and $\lambda z' \notin k[x_0, u_2, \ldots, u_n]$ it is follows $z' = 0$, and therefore $\varphi(z) = z$.

Consider, for example, the polynomial $z := x_4 x_0 - 4\, x_1\, x_3 + 3\, x_2^2$. Substituting

$$\begin{aligned}
&x_1 = -\lambda x_0,\\
&x_2 = u_2 + x_0 \lambda^2,\\
&x_3 = u_3 - 3\, u_2\, \lambda - x_0\, \lambda^3,\\
&x_4 = u_4 + 6 u_2\, \lambda^2 - 4\, u_3\, \lambda + x_4\, \lambda^4,
\end{aligned}$$

in $z$, after simplification we obtain $z = x_0\, u_4 + 3\, u_2^2$. On the other hand

$$\varphi(z) = \varphi(x_4\, x_0 - 4\, x_1\, x_3 + 3\, x_2^2) = x_0\, u_4 + 3\, u_2^2.$$

Thus $\varphi(z) = z$. $\square$

Recall now that a weight $\omega(z)$ of homogeneous isobaric invariant $z \in k[X]^{SL_2}$ is called the value $n \deg(z) - \omega(z)$, where $\omega$ is a integral function which takes each monomial $x_0^{\alpha_0}\, x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ to $\alpha_1 + 2\,\alpha_2 + \cdots + n\,\alpha_n$. From previous lemma it is follows that $\omega(z) = \omega_u(z)$ if only $z \in I_n \cap k[X]$. It is well known (Hilbert, [2] , p.38) that for an isobaric polynomial $z$, the conditions $d_1(z) = 0$ and $n \deg(z) = \omega(z)$ follow $d_2(z) = 0$. Thus each polynomial of $I_n \cap k[X]$ is invariant of $SL_2$. $\square$

## 4. Algorithm

By using the results of theorems 1, 2, in the cases $n \leq 6$ and $n = 8$, one may develope an effective algorithm for calculations of minimal generating sets of the invariant ring $R_n^{SL_2}$ and theirs syzygies. The main algorithm consist of several subsidiary algorithms. Note that we use the information about number of invariants, number of syzygies and theirs degree.

Let us go to the description of the algorithms.

**Algorithm (main)** MINGENSET(n, r, D).

**Input:** $n$ is degree of an binary form; $r$ is a number of homogeneous invariants which forms a minimal generating set of the invariant ring $R_n^{SL_2}$ and $D := \{s_1, s_2, \ldots, s_r\}$, $s_i \leq s_{i+1}$ is the set of their degries

**Output:** Minimal generating set of the invariant ring $R_n^{SL_2}$.

**begin**
$S := \{\emptyset\}$;
**for i from** $s_1$ **to** $s_r$ **do**
$I =$ INVARIANTS(n, i);
**for k from** $1$ **to** $nops(I)$ **do**
**if** MEMBER$(S, I_k)$ **then** $S := S$ **union** $\{I_k\}$ **end if;**
**end do;**
**end do;**
**return S;**
**end.**

**Algorithm** INVARIANTS$(n, d)$.

**Input:** $n$ is the degree of binary form;$d$ is a degree of an invariant.
**Output:** The set of linearly independed homogeneous invariants of degree $n$.
**begin**
$P :=$ POWERS$(n, d)$;
$F :=$ GPOL$(P)$;
$F1 :=$ DERPOL$(F)$;
$F2 :=$ COEFFS$(F1)$;
$F3 :=$ SYSTEM$(F2)$;
$I :=$ SOLVSUBS$(F3, F)$;
**return** $I$;
**end.**

**Algorithm** MEMBER$(S, F)$.

**Input:** $S$ is a set of homogeneous polynomials $\{f_1, f_2, \ldots, f_m\}$; $f$ a homogeneous polynomial.
**Output:** TRUE if $F \notin k[f_1, f_2, \ldots, f_m]$ and FALSE, if $F \in k[s_1, s_2, \ldots, s_m]$.
**begin**
$P :=$ GRAD$(S, f)$;
$F :=$ GPOL2$(P)$;
$F1 :=$ SUBS$(S, F)$
$F2 :=$ COEFFS$(F1)$;
$F3 :=$ SYSTEM$(F2)$;
$S :=$ ISSOLVABLE$(F3)$;
**return** $S$;
**end.**

**Algorithm** SYZYGIES$(S, r, D)$.

**Input:** $S$ is a set of homogeneous polynomials $\{f_1, f_2, \ldots, f_m\}$; $r$ is a number of syzygies and $D := \{s_1, s_2, \ldots, s_r\}$ is a set of their degries.
**Output:**The set of $r$ syzygies of the set $S$ .
**begin**
$S := \{\emptyset\}$;
**for i from** $1$ **to** $r$ **do**
$F :=$ POWERS2$(S, i)$;
$F1 :=$ GPOL2$(P)$
$F2 :=$ COEFFS$(F1)$
$F3 :=$ SYSTEM$(F)$
$F4 :=$ SOLVSYSTEM$(R, F)$
$S := S$ **union** $\{F4\}$;
**end do;**
**return S;**
**end.**

**Algorithm** POWERS$(n, d)$.

**Input:** $n$ is binary form degree; $d$ is a degree of an invariant.
**Output:** A set of solutions of the equation system

$$\begin{cases} \alpha_0 + \alpha_2 + \ldots + \alpha_n = d, \\ 2\alpha_2 + 3\alpha_3 + \ldots + n\alpha_n = \dfrac{n\,d}{2}, \end{cases} \quad \alpha_i \in \mathbb{Z}_+.$$

**Algorithm** POWERS2$(S, d)$.

**Input:** $S$ is set of homogeneous polynomials $\{f_1, f_2, \ldots, f_m\}$; $d$ is a degree of their sysygy.
**Output:** $P$ is a set of solutions of the equation

$$\alpha_1 \deg(f_1) + \alpha_2 \deg(f_2) + \ldots + \alpha_m \deg(f_m) = d, \alpha_i \in \mathbb{Z}_+.$$

**Algorithm** GRAD$(S, f)$.

**Input:** $S$ is a set of homogeneous polynomials $\{f_1, f_2, \ldots, f_m\}$; $f$ is a homogeneous polynomial.
**Output:** $P$ is set of solutions of the equation

$$\begin{cases} \alpha_1 \deg(f_1) + \alpha_2 \deg(f_2) + \ldots + \alpha_m \deg(f_m) = \deg(f), \\ \alpha_1 \omega(f_1) + \alpha_2 \omega(f_2) + \ldots + \alpha_m \omega(f_m) = \omega(f). \end{cases} \quad \alpha_i \in \mathbb{Z}_+.$$

**Algorithm** GPOL$(P)$.

**Input:** $P$ is a set of $n$-tuples $\{(\alpha_0, \alpha_2, \ldots, \alpha_n)\}$.
**Output:** $F$ is a "general" polinomial

$$F := \sum_{\alpha \in P} \beta_\alpha x_0^{\alpha_0} u_2^{\alpha_2} \cdots u_n^{\alpha_n}.$$

**Algorithm** GPOL2$(P)$.

**Input:** $P$ is a set of $m$-tuples $\{(\alpha_1, \alpha_2, \ldots, \alpha_m)\}$.
**Output:** $F$ is a "general" polinomial

$$F := \sum_{\alpha \in P} \beta_\alpha f_1^{\alpha_1} f_2^{\alpha_2} \cdots f_m^{\alpha_m}.$$

**Algorithm** DERPOL$(F)$.

**Input:** $F$ is a polynomial of $F \in k[x_0, u_2, \ldots, u_n]$.
**Output:** A result of the action of the derivation $d := x_0\, \hat{d}_2 - (n-1) u_2 \hat{d}_1$ applied to the polynomial $F$.

**Algorithm** SUBS$(S, F)$.

**Input:**$S$ is a set of homogeneous polynomials $\{f_1, f_2, \ldots, f_m\}$; $F$ is a polynomial of $F \in k[f_1, f_2, \ldots, f_m]$.
**Output:** A result of substituting of $S$ in the polynomial $F$.

**Algorithm** COEFFS$(F)$.

**Input:** $F$ is a polynomial of $k[x_0, u_2, \ldots, u_n]$.
**Output:** A set of coefficients of the polynomial $F$.

**Algorithm** SYSTEM$(F)$.

**Input:** $F$ a polynomial $F \in k[x_0, u_2, \dots, u_n]$.
**Output:** A system of linear homogeneous equations for the indeterminates $\beta_1, \beta_2, \dots$. We get the system by using the condition $F \equiv 0$.

**Algorithm** SOLVSYSTEM$(R, F)$.

**Input:** $R$ is a system of linear homogeneous equations; $F$ is a "general" polynomial of $k[f_1, f_2, \dots, f_m]$.
**Output:** A result of substituting of solutions of the sysytem $R$ in the polynomial $F$.

**Algorithm** SOLVSUBS$(R, F)$.

**Input:** $R$ is a system of linear homogeneous equations for the indeterminates $\beta$;
$F \in k[x_0, u_2, \dots, u_n]$.
**Output:** A vector space basis of solutions of the equation $d(F) = 0$.

**Algorithm** ISSOLVABLE$(R)$.

**Input:** $R$ is a system of linear homogeneous equations for the indeterminates $\beta_1, \beta_2, \dots$
**Output:** TRUE if the system $R$ has no solutions and FALSE if $R$ has solutions.

## 5. Examples

Below one may find differential equations and syzygies of the invariant ring $R_n^{SL_2}$ for $n \leq 6$ and $n = 8$. Also here is plased a minimal generating system of the invariants rings in the case $n < 5$. All calculations were done with Maple according to the above algorithms.

**n = 3.**

The differential equation for the invariant $u$, $u \in k[x_0, u_2, u_3]$ is as follow:

$$-3\, x_0^2\, \lambda \frac{\partial}{\partial x_0} u + (u_3\, x_0 + u_2\, x_0\, \lambda)\, (\frac{\partial}{\partial u_2} u) + (3\, u_3\, x_0 - 6\, u_2{}^2)\, (\frac{\partial}{\partial u_3} u) = 0,$$

or

$$d(u) = x_0\, \hat{d}_2(u) - 2u_2 \hat{d}_1(u) = 0.$$

The invariant ring $R_3^{SL_3}$ generated by one invariant $f_4$ of degree four. The subscript in $f_i$ means a degree of the polynomial $f_i$.The weight of $f_4$ is equal $\dfrac{3 \cdot 4}{2} = 6$. The system of equations

$$\begin{cases} \alpha_0 + \alpha_2 + \alpha_3 = 4, \\ 2\alpha_2 + 3\alpha_3 = 6, \end{cases}$$

in $\mathbb{Z}_+$ has only the following two solutions – $(1, 3, 0)$ i $(2, 0, 2)$. Then we find an invariant in the form $f_4 = \beta_1 x_0 u_2^3 + \beta_2 x_0^2 u_3^2$. We have

$$d(f_4) = x_0\, \hat{d}_2(f_4) - 2u_2 \hat{d}_1(f_4) = x_0(\beta_1 3x_0 u_2^2 u_3) - 2u_2(\beta_2 6x_0^2 u_2 u_3) = (3\beta_1 - 12\beta_2) x_0^2 u_2^2 u_3 = 0.$$

This implyies $3\beta_1 - 12\beta_2 = 0$, or $\beta_1 = 4\beta_2$. Thus $f_4 = 4x_0 u_2^3 + x_0^2 u_3^2$ and

$$R_3^{SL_2} = k[f_4].$$

Going back to indeterminates $x_0, x_1, x_2, x_3$ we get

$$f_4 = 4\, x_0 x_2^3 - 3x_1^2 x_2^2 + t^2\, x_3^2 - 6\, x_0 x_1 x_2 x_3 + 4\, x_1^3\, x_3.$$

**n = 4.**

The differential equation for the invariant $u$, $u \in k[x_0, u_2, u_3, u_4]$ is as follow:

$$-4\,x_0^2\,\lambda\,(\frac{\partial}{\partial t}\,u)+2\,u_3\,x_0\,(\frac{\partial}{\partial u_2}\,u)+(x_0\,u_4+2\,\lambda\,x_0\,u_3-9\,{u_2}^2)\,(\frac{\partial}{\partial u_3}\,u)+$$

$$+(4\,u_4\,x_0\,\lambda-12\,u_2\,u_3)\,(\frac{\partial}{\partial u_4}\,u)=0$$

The invariant ring $R_4^{SL_2}$ generated by two invariants $f_2, f_3$. Those invariants one may find in the same way as it were done for the case $n=3$..

$$R_4^{SL_2}=k[f_2,f_3],\,f_2:=t\,u_4+3\,{u_2}^2,\,f_3:={u_2}^3-t\,u_2\,u_4+t\,{u_3}^2$$

**n = 5**.

The differential equation is as follow

$$-5\,x_0^2\,\lambda\,(\frac{\partial}{\partial x_0}\,u)+(3\,u_3\,x_0-u_2\,x_0\,\lambda)\,(\frac{\partial}{\partial u_2}\,u)+(2\,x_0\,u_4+\lambda\,x_0\,u_3-12\,{u_2}^2)\,(\frac{\partial}{\partial u_3}\,u)+$$

$$+(u_5\,x_0+3\,u_4\,x_0\,\lambda-16\,u_2\,u_3)\,(\frac{\partial}{\partial u_4}\,u)+(5\,u_5\,x_0\,\lambda-20\,u_2\,u_4)\,(\frac{\partial}{\partial u_5}\,u)=0$$

The invariant ring $R_5^{SL_2}$ generated by four invariants $f_4, f_8, f_{12}, f_{18}$.

$$R_5^{SL_2}=k[f_4,f_8,f_{12},f_{18}],$$

There exists the single sygyzy:

$$1296\,f_{18}^2=-48\,{f_{12}}^3+{f_4}^5\,{f_8}^2-6\,{f_4}^3\,{f_8}^3+9\,f_4\,{f_8}^4-2\,{f_4}^4\,f_8\,f_{12}-18\,{f_4}^2\,{f_8}^2\,f_{12}+72\,{f_8}^3\,f_{12}+$$
$$+\,{f_4}^3\,{f_{12}}^2+72\,f_4\,f_8\,{f_{12}}^2$$

**n = 6**.

The diffrential equation is as follows

$$-6\,x_0^2\,\lambda\,\frac{\partial}{\partial x_0}\,u+(4\,u_3\,x_0-2\,u_2\,x_0\,\lambda)\,\frac{\partial}{\partial u_2}\,u+$$

$$+(3\,u_4\,x_0-15\,{u_2}^2)\,\frac{\partial}{\partial u_3}\,u+(2\,u_5\,x_0+2\,u_4\,x_0\,\lambda-20\,u_2\,u_3)\,\frac{\partial}{\partial u_4}\,u$$

$$+(u_6\,x_0+4\,u_5\,x_0\,\lambda-25\,u_2\,u_4)\,\frac{\partial}{\partial u_5}\,u+(6\,u_6\,x_0\,\lambda-30\,u_2\,u_5)\,\frac{\partial}{\partial u_6}\,u=0$$

The invariant ring $R_6^{SL_2}$ generated by five invariants $f_2, f_4, f_6, f_{10}, f_{15}$.

$$R_6^{SL_2}=k[f_2,f_4,f_6,f_{10},f_{15}],$$

The exists the single sygyzy

$$
\begin{aligned}
&640722656250000\, f_{15}{}^2 = 4556250000\, f_2\, f_4{}^3\, f_6\, f_{10} - 6378750000000\, f_2{}^2\, f_4\, f_6{}^2\, f_{10} \\
&+ 89606250000\, f_2{}^5\, f_4\, f_6\, f_{10} + 51637500000\, f_2{}^3\, f_4{}^2\, f_6\, f_{10} + 160692\, f_2{}^{11}\, f_4{}^2 \\
&+ 864\, f_2{}^3\, f_4{}^6 + 10242\, f_2{}^5\, f_4{}^5 + 54820\, f_2{}^7\, f_4{}^4 + 133195\, f_2{}^9\, f_4{}^3 + 94864\, f_2{}^{13}\, f_4 \\
&+ 27\, f_2\, f_4{}^7 - 20344800\, f_2{}^{12}\, f_6 - 14175\, f_4{}^6\, f_6 + 6473250000\, f_2{}^9\, f_6{}^2 \\
&- 817003125000\, f_2{}^6\, f_6{}^3 + 379687500\, f_4{}^3\, f_6{}^3 + 30691406250000\, f_2{}^3\, f_6{}^4 \\
&- 158760000\, f_2{}^{10}\, f_{10} - 1822500\, f_4{}^5\, f_{10} - 3189375000000\, f_2{}^5\, f_{10}{}^2 \\
&- 71191406250000\, f_6{}^5 + 2562890625000000\, f_{10}{}^3 + 21952\, f_2{}^{15} \\
&+ 14207062500\, f_2{}^7\, f_4\, f_6{}^2 - 540675\, f_2{}^2\, f_4{}^5\, f_6 + 9055125000\, f_2{}^5\, f_4{}^2\, f_6{}^2 \\
&+ 1382062500\, f_2{}^3\, f_4{}^3\, f_6{}^2 + 60750000\, f_2\, f_4{}^4\, f_6{}^2 - 895345312500\, f_2{}^4\, f_4\, f_6{}^3 \\
&- 949218750000\, f_2\, f_4\, f_6{}^4 - 510300000\, f_2{}^8\, f_4\, f_{10} - 580162500\, f_2{}^6\, f_4{}^2\, f_{10} \\
&- 39487500\, f_2{}^2\, f_4{}^4\, f_{10} + 42525000000\, f_2{}^7\, f_6\, f_{10} - 6037031250000\, f_2{}^4\, f_6{}^2\, f_{10} \\
&- 77962500000\, f_2{}^2\, f_4{}^2\, f_6{}^3 - 266287500\, f_2{}^4\, f_4{}^3\, f_{10} + 187945312500000\, f_2{}^2\, f_6\, f_{10}{}^2 \\
&- 66628800\, f_2{}^{10}\, f_4\, f_6 - 78315750\, f_2{}^8\, f_4{}^2\, f_6 - 38636625\, f_2{}^6\, f_4{}^3\, f_6 - 7131375\, f_2{}^4\, f_4{}^4\, f_6 \\
&- 341718750000\, f_4{}^2\, f_6{}^2\, f_{10} + 427148437500000\, f_2\, f_6{}^3\, f_{10} \\
&- 3531093750000\, f_2{}^3\, f_4\, f_{10}{}^2 - 341718750000\, f_2\, f_4{}^2\, f_{10}{}^2 \\
&- 25628906250000\, f_4\, f_6\, f_{10}{}^2
\end{aligned}
$$

**n = 8**.

The differential equations

$$
\begin{aligned}
&-8\, x_0^2\, \lambda\, (\frac{\partial}{\partial x_0} u) + (6\, u_3\, x_0 - 4\, u_2\, x_0\, \lambda)\, (\frac{\partial}{\partial u_2} u) + (5\, x_0\, u_4 - 2\, \lambda\, x_0\, u_3 - 21\, u_2{}^2)\, (\frac{\partial}{\partial u_3} u) \\
&+ (4\, u_5\, x_0 - 28\, u_2\, u_3)\, (\frac{\partial}{\partial u_4} u) + (3\, u_6\, x_0 + 2\, u_5\, x_0\, \lambda - 35\, u_2\, u_4)\, (\frac{\partial}{\partial u_5} u) \\
&+ (2\, u_7\, x_0 + 4\, u_6\, x_0\, \lambda - 42\, u_2\, u_5)\, (\frac{\partial}{\partial u_6} u) + (u_8\, x_0 + 6\, u_7\, x_0\, \lambda - 49\, u_2\, u_6)\, (\frac{\partial}{\partial u_7} u) \\
&+ (8\, u_8\, x_0\, \lambda - 56\, u_2\, u_7)\, (\frac{\partial}{\partial u_8} u) = 0
\end{aligned}
$$

The invariant ring $R_8^{SL_2}$ generated by nine invariants $f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}$. There exists the five sygyzies

$$
\begin{aligned}
&100822400\, f_5\, f_4\, f_3\, f_2{}^2 + 2318131200\, f_6\, f_5\, f_3\, f_2 + 10838016\, f_6{}^2\, f_4 - 94832640\, f_9\, f_7 \\
&- 67436544\, f_{10}\, f_6 - 702464\, f_{10}\, f_2{}^3 + 127573232640\, f_5{}^2\, f_2{}^3 + 336672\, f_3{}^2\, f_2{}^5 \\
&- 643328\, f_6\, f_2{}^5 + 1628\, f_4\, f_2{}^6 + 4070\, f_4{}^2\, f_2{}^4 - 1303680\, f_8\, f_2{}^4 - 31911936\, f_6{}^2\, f_2{}^2 \\
&- 8996\, f_4{}^3\, f_2{}^2 - 8956416\, f_3{}^4\, f_2{}^2 - 12117504\, f_4\, f_3{}^4 - 8768289116160\, f_5{}^2\, f_3{}^2 \\
&+ 48470016\, f_{10}\, f_3{}^2 - 2808960\, f_8\, f_4{}^2 + 12324702781440\, f_6\, f_5{}^2 + 411936\, f_4{}^2\, f_3{}^2\, f_2 \\
&- 1369804800\, f_5\, f_3{}^3\, f_2 + 4741632000\, f_9\, f_5\, f_2 - 253848161280\, f_5{}^2\, f_4\, f_2 \\
&+ 1404928\, f_{10}\, f_4\, f_2 - 88670400\, f_5\, f_4{}^2\, f_3 + 8851046400\, f_8\, f_5\, f_3 - 8467200\, f_9\, f_4\, f_3 \\
&- 58329600\, f_7\, f_5\, f_4 + 71688960\, f_9\, f_3\, f_2{}^2 + 4112640\, f_8\, f_4\, f_2{}^2 + 16181760\, f_7\, f_5\, f_2{}^2 \\
&+ 33078528\, f_6\, f_3{}^2\, f_2{}^2 + 1650432\, f_6\, f_4\, f_2{}^3 - 748608\, f_4\, f_3{}^2\, f_2{}^3 - 19176640\, f_5\, f_3\, f_2{}^4 \\
&+ 9069312\, f_6\, f_4\, f_3{}^2 - 1007104\, f_6\, f_4{}^2\, f_2 + 4305\, f_4{}^4 - 1007\, f_2{}^8 + 189665280\, f_8{}^2 = 0
\end{aligned}
$$

$$
\begin{aligned}
&1693440\, f_5\, {f_3}^2\, f_4\, f_2 + 188160\, f_8\, f_3\, f_4\, f_2 + 23802240\, f_6\, f_5\, f_4\, f_2 + 412048\, f_6\, f_3\, {f_2}^2\, f_4 \\
&- 1011548160\, {f_6}^2\, f_5 + 284497920\, f_9\, f_8 - 167160\, f_5\, {f_2}^6 + 88200\, f_5\, {f_4}^3 + 70560\, f_9\, {f_2}^4 \\
&+ 4913121024000\, {f_5}^3\, f_2 + 85680\, {f_4}^2\, {f_3}^3 - 1485711360\, f_5\, {f_3}^4 + 2184\, f_7\, {f_2}^5 \\
&- 98262420480\, f_7\, {f_5}^2 + 526848\, f_{10}\, f_7 + 123312\, {f_3}^3\, {f_2}^4 + 10536960\, {f_3}^5\, f_2 \\
&- 31610880\, f_8\, {f_3}^3 - 2071\, f_3\, {f_2}^7 - 493920\, f_9\, {f_4}^2 + 1818880\, f_5\, {f_3}^2\, {f_2}^3 + 4757\, f_3\, {f_2}^5\, f_4 \\
&+ 74889467520\, {f_5}^2\, f_3\, {f_2}^2 - 4368\, f_7\, {f_2}^3\, f_4 - 26342400\, f_{10}\, f_5\, f_2 + 56448\, f_7\, f_4\, {f_3}^2 \\
&+ 615\, {f_4}^3\, f_3\, f_2 + 2184\, f_7\, {f_4}^2\, f_2 - 115080\, f_6\, {f_4}^2\, f_3 + 272832\, f_7\, {f_2}^2\, f_6 - 272832\, f_7\, f_6\, f_4 \\
&- 208992\, {f_3}^3\, {f_2}^2\, f_4 + 2402426880\, f_6\, f_5\, {f_3}^2 - 188160\, f_8\, f_3\, {f_2}^3 + 47040\, f_{10}\, f_4\, f_3 \\
&+ 9009100800\, f_9\, f_5\, f_3 + 423360\, f_9\, {f_2}^2\, f_4 - 296968\, f_6\, f_3\, {f_2}^4 + 19192320\, f_8\, {f_2}^2\, f_5 \\
&- 23802240\, f_6\, f_5\, {f_2}^3 - 10382198400\, {f_5}^2\, f_4\, f_3 - 56448\, f_7\, {f_3}^2\, {f_2}^2 - 398272\, f_{10}\, f_3\, {f_2}^2 \\
&- 343560\, f_5\, {f_4}^2\, {f_2}^2 + 94832640\, f_9\, f_6\, f_2 - 50803200\, f_8\, f_5\, f_4 + 10536960\, {f_6}^2\, f_3\, f_2 \\
&+ 31610880\, f_8\, f_6\, f_3 - 3301\, {f_4}^2\, f_3\, {f_2}^3 + 422520\, f_5\, f_4\, {f_2}^4 - 94832640\, f_9\, {f_3}^2\, f_2 \\
&- 21073920\, {f_3}^3\, f_6\, f_2 = 0
\end{aligned}
$$

$$
\begin{aligned}
&-609623613760\, f_6\, f_5\, f_3\, {f_2}^2 - 14374530240\, f_7\, f_5\, f_4\, f_2 - 19882914240\, f_5\, {f_4}^2\, f_3\, f_2 \\
&+ 738439705920\, f_6\, f_5\, f_4\, f_3 + 302779545600\, f_8\, f_5\, f_3\, f_2 + 17615939040\, f_9\, f_4\, f_3\, f_2 \\
&+ 8715924784\, f_6\, f_4\, {f_3}^2\, f_2 + 26684913920\, f_5\, f_4\, f_3\, {f_2}^3 + 1053441\, {f_4}^4\, f_2 - 526187\, f_4\, {f_2}^7 \\
&- 63819497\, f_6\, {f_2}^6 - 2484310976\, {f_6}^2\, {f_2}^3 - 100742040\, f_{10}\, {f_2}^4 + 2105815\, {f_4}^2\, {f_2}^5 \\
&+ 18064191090960\, {f_5}^2\, {f_2}^4 - 2633069\, {f_4}^3\, {f_2}^3 + 19792514\, {f_3}^2\, {f_2}^6 \\
&+ 2158397472\, {f_3}^4\, {f_2}^3 - 281048376\, f_8\, {f_2}^5 - 807740962560\, {f_6}^2\, {f_3}^2 + 29521212\, {f_4}^3\, {f_3}^2 \\
&- 3713751552\, f_{10}\, f_8 + 752034689280\, f_9\, {f_3}^3 + 724181552640\, f_6\, {f_3}^4 \\
&+ 48553512\, f_{10}\, {f_4}^2 - 8945581947120\, {f_5}^2\, {f_4}^2 - 58978983\, f_6\, {f_4}^3 \\
&+ 692651801963520\, f_8\, {f_5}^2 + 6488133120\, {f_8}^2\, f_2 + 10389777029452800\, {f_5}^3\, f_3 \\
&- 1331385664\, {f_6}^2\, f_4\, f_2 - 5974094112\, f_4\, {f_3}^4\, f_2 + 1978375054064640\, f_6\, {f_5}^2\, f_2 \\
&- 1353939100442880\, {f_5}^2\, {f_3}^2\, f_2 - 553598136\, f_8\, {f_4}^2\, f_2 - 10972312064\, f_{10}\, f_6\, f_2 \\
&+ 7538185088\, f_{10}\, {f_3}^2\, f_2 - 3244066560\, f_9\, f_7\, f_2 - 55706273280\, f_{10}\, f_5\, f_3 \\
&- 1002712919040\, f_9\, f_6\, f_3 - 437692147200\, f_9\, f_5\, f_4 + 23144630208\, f_8\, f_6\, f_4 \\
&+ 594200248320\, f_7\, f_6\, f_5 + 265657835520\, f_9\, f_5\, {f_2}^2 + 6743136960\, f_7\, f_5\, {f_2}^3 \\
&- 25622422\, {f_4}^2\, {f_3}^2\, {f_2}^2 - 32010116547360\, {f_5}^2\, f_4\, {f_2}^2 + 428191980160\, f_5\, {f_3}^3\, {f_2}^2 \\
&+ 834646512\, f_8\, f_4\, {f_2}^3 - 6168849120\, f_9\, f_3\, {f_2}^3 - 23691304\, f_4\, {f_3}^2\, {f_2}^4 \\
&- 1084531504\, f_6\, {f_3}^2\, {f_2}^3 - 23144630208\, f_8\, f_6\, {f_2}^2 + 15650810112\, f_8\, {f_3}^2\, {f_2}^2 \\
&- 496714270080\, f_5\, f_4\, {f_3}^3 - 427081428480\, f_7\, f_5\, {f_3}^2 - 15650810112\, f_8\, f_4\, {f_3}^2 \\
&+ 134526203\, f_6\, f_4\, {f_2}^4 - 8073898560\, f_5\, f_3\, {f_2}^5 - 11727723\, f_6\, {f_4}^2\, {f_2}^2 \\
&+ 179378416\, f_{10}\, f_4\, {f_2}^2 - 1002712919040\, {f_9}^2 + 297100124160\, {f_6}^3 \\
&- 213540714240\, {f_3}^6 = 0
\end{aligned}
$$

$$
\begin{aligned}
&-33148029168672082560\, f_9\, {f_3}^3\, f_2 - 16151199565724160000\, f_{10}\, f_5\, f_3\, f_2 \\
&+ 159410369241955440\, f_9\, f_4\, f_3\, {f_2}^2 + 6616249713784757760\, f_5\, f_4\, {f_3}^3\, f_2 \\
&+ 749107528070052480\, f_7\, f_6\, f_3\, {f_2}^2 + 183429726867238152\, f_6\, f_4\, {f_3}^2\, {f_2}^2 \\
&+ 243683386620972480\, f_8\, f_4\, {f_3}^2\, f_2 + 156955879038822080\, f_5\, f_4\, f_3\, {f_2}^4 \\
&+ 33282997663046261760\, f_9\, f_6\, f_3\, f_2 - 161855888224392040\, f_5\, {f_4}^2\, f_3\, {f_2}^2 \\
&+ 21602829224597958400\, f_6\, f_5\, f_3\, {f_2}^3 - 749107528070052480\, f_7\, f_6\, f_4\, f_3 \\
&+ 36993024884145012480\, f_8\, f_5\, f_3\, {f_2}^2 + 599655040942776\, f_7\, {f_4}^2\, f_3\, f_2 \\
&- 1199310081885552\, f_7\, f_4\, f_3\, {f_2}^3 - 14685726803748000\, f_7\, f_5\, f_4\, {f_2}^2 \\
&- 43002814322146156800\, f_8\, f_5\, f_4\, f_3 - 47900448277296384000\, f_7\, f_6\, f_5\, f_2 \\
&- 287852667962852160\, f_8\, f_6\, f_4\, f_2 + 17247956632804800000\, f_9\, f_5\, f_4\, f_2 \\
&+ 47000658314801856000\, f_7\, f_5\, {f_3}^2\, f_2 + 461121658725\, {f_4}^5 \\
&+ 1015806871869142404473856000\, {f_5}^4 + 29800796718210560\, {f_{10}}^2 \\
&- 191358336189835\, f_6\, {f_4}^2\, {f_2}^3 + 225505508636567040\, f_{10}\, f_8\, f_2 \\
&- 1318757106260160\, f_9\, f_7\, {f_2}^2 - 164742908464499904\, f_6\, f_5\, f_4\, f_3\, f_2 \\
&+ 98229371056648634880\, f_9\, f_8\, f_3 + 15498776442828672\, f_7\, f_4\, {f_3}^3 \\
&+ 10842892786412887603200\, f_6\, f_5\, {f_3}^3 + 438359081750268000\, f_5\, {f_4}^3\, f_3 \\
&+ 10033182679464416640\, {f_6}^2\, {f_3}^2\, f_2 + 3437021584237713561600000\, {f_5}^3\, f_3\, f_2 \\
&- 627311151362564894400\, {f_5}^2\, f_4\, {f_3}^2 - 48212408500735360\, f_{10}\, f_6\, f_4 \\
&+ 90011710269290880\, f_{10}\, f_6\, {f_2}^2 + 321005969107880\, f_{10}\, {f_4}^2\, f_2 \\
&+ 34609880419465\, f_6\, {f_4}^3\, f_2 + 88787506880235584736000\, f_6\, {f_5}^2\, f_4 \\
&+ 80462151139168512000\, f_9\, f_6\, f_5 - 13410358523194752000\, f_8\, f_7\, f_5 \\
&+ 688896004392960\, f_8\, f_4\, {f_2}^4 + 30569553160344277632000\, f_9\, f_5\, {f_3}^2 \\
&+ 22089673331178808320\, f_8\, f_6\, {f_3}^2 - 12881298117334375680\, f_6\, {f_3}^4\, f_2 \\
&- 243683386620972480\, f_8\, {f_3}^2\, {f_2}^3 - 115637361265270992\, f_6\, {f_3}^2\, {f_2}^4 \\
&- 56542404757067308800\, f_5\, {f_3}^3\, {f_2}^3 + 1081600974673338\, f_4\, {f_3}^2\, {f_2}^5 \\
&- 522923155336869\, {f_4}^2\, {f_3}^2\, {f_2}^3 + 599655040942776\, f_7\, f_3\, {f_2}^5 \\
&- 58692356043286828800\, {f_5}^2\, {f_4}^2\, f_2 - 643416702093950400\, {f_6}^2\, f_4\, {f_2}^2 \\
&- 1345820594938680\, f_8\, {f_4}^2\, {f_2}^2 - 1630010402558056639680000\, f_6\, {f_5}^2\, {f_2}^2 \\
&- 407179965634866434304000\, f_8\, {f_5}^2\, f_2 - 928607984430000\, f_7\, f_5\, {f_4}^2 \\
&+ 287852667962852160\, f_8\, f_6\, {f_2}^3 + 2633797791953079360000\, {f_5}^2\, f_4\, {f_2}^3 \\
&+ 10826186455659456000\, f_9\, f_5\, {f_2}^3 - 138390688033360\, f_{10}\, f_4\, {f_2}^3 \\
&- 39361555264348800\, f_9\, f_7\, f_4 - 4408277051598000\, f_7\, f_5\, {f_2}^4 \\
&+ 106072821789875\, f_6\, f_4\, {f_2}^5 - 176022826904160768\, f_{10}\, {f_3}^2\, {f_2}^2 \\
&- 171200685583289880\, f_9\, {f_4}^2\, f_3 + 144655246799734272\, f_{10}\, f_7\, f_3 \\
&- 27404311433852939550720\, f_7\, {f_5}^2\, f_3 - 15498776442828672\, f_7\, {f_3}^3\, {f_2}^2 \\
&- 76155414888852768\, f_4\, {f_3}^4\, {f_2}^2 + 330471922211204704268800\, {f_5}^2\, {f_3}^2\, {f_2}^2 \\
&+ 287765186078832000\, f_{10}\, f_4\, {f_3}^2 - 477697537621911600\, f_6\, {f_4}^2\, {f_3}^2 \\
&- 4227300096275284000\, f_5\, f_3\, {f_2}^6 - 490068750472740042240\, {f_6}^2\, f_5\, f_3 \\
&+ 418242341009984400\, f_9\, f_3\, {f_2}^4 + 293380970594082000\, {f_4}^2\, {f_3}^4 \\
&- 558677819336469\, {f_3}^2\, {f_2}^7 - 620258472592500887040\, f_5\, {f_3}^5 \\
&- 15384494069581432320\, f_8\, {f_3}^4 + 5053165179535872000\, {f_8}^2\, {f_2}^2 \\
&- 110045402041336134912000\, f_{10}\, {f_5}^2 - 2395022413864819200\, {f_6}^3\, f_2 \\
&+ 36806011909556568\, {f_3}^4\, {f_2}^4 - 1383364976175\, {f_4}^3\, {f_2}^4 + 922243317450\, {f_4}^2\, {f_2}^6 \\
&+ 227707288714920\, f_8\, {f_2}^6 - 40021826529968640000\, {f_8}^2\, f_4 \\
&+ 3571073248784776000\, {f_6}^2\, {f_2}^4 + 151094916255080\, f_{10}\, {f_2}^5 \\
&- 67051792615973760000\, f_8\, {f_6}^2 + 50675633980495\, f_6\, {f_2}^7 \\
&- 27618370448837952000\, {f_5}^2\, {f_2}^5 + 5398739774967168000\, {f_9}^2\, f_2 \\
&+ 429217301830800\, f_8\, {f_4}^3 + 186196318016592800\, {f_6}^2\, {f_4}^2
\end{aligned}
$$

$$
\begin{aligned}
&-376564832\, f_{10}\, f_3\, {f_2}^3 + 69222949461120\, {f_5}^2\, f_3\, {f_2}^3 - 77762764800\, f_8\, f_5\, {f_3}^2 \\
&+ 4793428080\, f_9\, f_4\, {f_3}^2 + 1131285120\, f_6\, f_4\, {f_3}^3 + 11070393600\, f_7\, f_6\, {f_3}^2 \\
&- 767151000\, f_5\, {f_4}^2\, {f_3}^2 - 2118060000\, f_6\, f_5\, {f_4}^2 - 156548712\, {f_3}^3\, {f_2}^3\, f_4 \\
&- 411521160\, f_8\, f_3\, {f_2}^4 + 395371200\, f_8\, f_7\, {f_2}^2 + 267446865\, f_5\, f_4\, {f_2}^5 \\
&- 186747045\, f_5\, {f_4}^2\, {f_2}^3 - 26327253120\, f_{10}\, f_5\, {f_2}^2 + 217319\, f_7\, {f_4}^2\, {f_2}^2 \\
&- 257277368\, f_6\, f_3\, {f_2}^5 - 20774130720\, f_6\, f_5\, {f_2}^4 - 2325763\, f_7\, {f_2}^4\, f_4 \\
&+ 263393312\, f_7\, {f_2}^3\, f_6 - 54495168\, f_7\, {f_3}^2\, {f_2}^3 - 7178672200\, f_5\, {f_3}^2\, {f_2}^4 \\
&- 21424717440\, {f_3}^3\, f_6\, {f_2}^2 + 285866280\, f_9\, {f_2}^3\, f_4 - 7170770880\, f_8\, {f_2}^3\, f_5 \\
&+ 2804152\, f_3\, {f_2}^6\, f_4 - 1402076\, {f_4}^2\, f_3\, {f_2}^4 + 10390482480\, {f_6}^2\, f_3\, {f_2}^2 \\
&+ 56205696960\, f_9\, f_6\, {f_2}^2 - 1558341120\, f_{10}\, f_6\, f_3 + 2372227200\, f_{10}\, f_5\, f_4 \\
&+ 2135004480\, f_9\, f_6\, f_4 - 395371200\, f_8\, f_7\, f_4 + 132844723200\, f_8\, f_6\, f_5 \\
&- 74127377520\, f_9\, {f_3}^2\, {f_2}^2 + 60155220\, {f_4}^2\, {f_3}^3\, f_2 + 16489872000\, f_9\, f_7\, f_3 \\
&+ 508621568\, f_{10}\, f_7\, f_2 - 243766320\, {f_6}^2\, f_4\, f_3 - 93202449607680\, f_7\, {f_5}^2\, f_2 \\
&- 742239644160\, f_5\, {f_3}^4\, f_2 + 35349075\, f_5\, {f_4}^3\, f_2 - 389822549760\, {f_6}^2\, f_5\, f_2 \\
&- 19446900480\, f_8\, {f_3}^3\, f_2 + 1074878676480\, f_6\, f_5\, {f_3}^2\, f_2 + 339403088\, f_6\, f_3\, {f_2}^3\, f_4 \\
&+ 21040314400\, f_7\, f_5\, f_3\, {f_2}^2 + 22892190720\, f_6\, f_5\, f_4\, {f_2}^2 + 11328061920\, f_5\, {f_3}^2\, f_4\, {f_2}^2 \\
&+ 445542960\, f_8\, f_3\, f_4\, {f_2}^2 - 82125720\, f_6\, {f_4}^2\, f_3\, f_2 + 38340960\, f_{10}\, f_4\, f_3\, f_2 \\
&+ 54495168\, f_7\, f_4\, {f_3}^2\, f_2 + 19446900480\, f_8\, f_6\, f_3\, f_2 - 12276129600\, f_8\, f_5\, f_4\, f_2 \\
&- 263393312\, f_7\, f_6\, f_4\, f_2 - 7470423273600\, {f_5}^2\, f_4\, f_3\, f_2 + 5382096652800\, f_9\, f_5\, f_3\, f_2 \\
&- 15556548000\, f_7\, f_5\, f_4\, f_3 + 175022104320\, f_9\, f_8\, f_2 - 316566180\, f_9\, {f_4}^2\, f_2 \\
&+ 286795093939200\, f_6\, {f_5}^2\, f_3 - 34021800\, f_8\, {f_4}^2\, f_3 - 5535196800\, f_7\, {f_3}^4 \\
&- 1402076\, f_3\, {f_2}^8 + 96393492\, {f_3}^3\, {f_2}^5 - 116048895\, f_5\, {f_2}^7 \\
&+ 4851583356211200\, {f_5}^3\, {f_2}^2 + 11024729520\, {f_3}^5\, {f_2}^2 + 1478069\, f_7\, {f_2}^6 \\
&+ 30699900\, f_9\, {f_2}^5 - 878013360\, f_4\, {f_3}^5 - 281791198886400\, {f_5}^2\, {f_3}^3 + 630375\, f_7\, {f_4}^3 \\
&+ 2427870161203200\, f_9\, {f_5}^2 - 433548243072000\, {f_5}^3\, f_4 - 11070393600\, {f_7}^2\, f_5 \\
&- 5535196800\, f_7\, {f_6}^2 - 13284472320\, f_{10}\, f_9 + 231436800\, {f_8}^2\, f_3 + 1535197440\, f_{10}\, {f_3}^3 \\
&= 0
\end{aligned}
$$

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## Appendix

The generating invariants $f_4, f_8, f_{12}, f_{18}$, in the case $n=5$. Here we denote the variable $x_0$ by $t$.

$f_4 := t^2\, u_5^{\,2} + 4\, t\, u_2\, u_3\, u_5 + 48\, u_4\, u_2^{\,3} - 32\, u_2^{\,2}\, u_3^{\,2} + 16\, t\, u_2\, u_4^{\,2} - 12\, t\, u_3^{\,2}\, u_4,$

$f_8 := 22\, t^2\, u_2^{\,2}\, u_4^{\,4} + 8\, u_2^{\,4}\, u_3^{\,4} + 18\, u_2^{\,6}\, u_4^{\,2} - 24\, u_2^{\,5}\, u_3^{\,2}\, u_4 - 38\, t\, u_2^{\,4}\, u_4^{\,3} + 18\, t^2\, u_3^{\,4}\, u_4^{\,2} -$
$- 27\, t^2\, u_5\, u_3^{\,5} - 2\, t^3\, u_4^{\,5} + 93\, t\, u_2^{\,4}\, u_3\, u_4\, u_5 - 34\, t^2\, u_2^{\,2}\, u_3\, u_5\, u_4^{\,2} + 12\, t^2\, u_2^{\,3}\, u_5^{\,2}\, u_4 +$
$+ 78\, t^2\, u_2\, u_3^{\,3}\, u_5\, u_4 - 27\, t\, u_5^{\,2}\, u_2^{\,5} - 21\, t^2\, u_2^{\,2}\, u_5^{\,2}\, u_3^{\,2} - 48\, t^2\, u_2\, u_3^{\,2}\, u_4^{\,3} + 8\, t\, u_2^{\,3}\, u_3^{\,2}\, u_4^{\,2}$
$- t^3\, u_2\, u_5^{\,2}\, u_4^{\,2} + 6\, t\, u_2^{\,2}\, u_3^{\,4}\, u_4 + t^3\, u_2\, u_3\, u_5^{\,3} - 42\, t\, u_2^{\,3}\, u_5\, u_3^{\,3} - 3\, t^3\, u_5^{\,2}\, u_3^{\,2}\, u_4 + 5\, t^3\, u_3\, u_5\, u_4^{\,3}$

$f_{12} := 252\, t^2\, u_3^{\,2}\, u_5^{\,2}\, u_4\, u_2^{\,5} - 2010\, t\, u_2^{\,6}\, u_5\, u_3^{\,3}\, u_4 - 470\, t^2\, u_2^{\,4}\, u_5\, u_3^{\,3}\, u_4^{\,2}$
$+ 180\, t^3\, u_2^{\,2}\, u_5^{\,2}\, u_3^{\,4}\, u_4 + 4\, t^4\, u_2\, u_4^{\,7} - 54\, t^4\, u_3\, u_2\, u_5\, u_4^{\,5} + 80\, t^4\, u_2\, u_5^{\,2}\, u_3^{\,2}\, u_4^{\,3}$
$- 243\, t^2\, u_3^{\,10} - 360\, t^3\, u_3\, u_2^{\,3}\, u_5\, u_4^{\,4} - 348\, t^2\, u_5\, u_3^{\,5}\, u_4\, u_2^{\,3} + 890\, t^3\, u_2^{\,2}\, u_3^{\,3}\, u_5\, u_4^{\,3}$
$+ 1180\, t^2\, u_2^{\,5}\, u_3\, u_5\, u_4^{\,3} - 270\, t^3\, u_2^{\,3}\, u_5^{\,2}\, u_3^{\,2}\, u_4^{\,2} + 9\, t^3\, u_2^{\,5}\, u_5^{\,4} - 360\, t\, u_2^{\,3}\, u_3^{\,8}$
$+ 664\, t^2\, u_4^{\,5}\, u_2^{\,5} - 2\, t^5\, u_5^{\,2}\, u_4^{\,5} + 112\, t^3\, u_2^{\,3}\, u_4^{\,6} - 360\, u_2^{\,8}\, u_5\, u_3^{\,3} + 9\, t^4\, u_5^{\,3}\, u_3^{\,5}$
$- 1680\, t\, u_4^{\,4}\, u_2^{\,7} - 90\, t^3\, u_3^{\,6}\, u_4^{\,3} + 1800\, u_2^{\,7}\, u_3^{\,4}\, u_4 - 3\, t^4\, u_3^{\,2}\, u_4^{\,6} - 2475\, u_2^{\,8}\, u_3^{\,2}\, u_4^{\,2}$
$+ 2520\, t\, u_2^{\,7}\, u_3\, u_5\, u_4^{\,2} + 900\, u_2^{\,9}\, u_4^{\,3} - t^5\, u_2\, u_5^{\,4}\, u_4^{\,2} + t^5\, u_3\, u_2\, u_5^{\,5} + 422\, t\, u_5\, u_3^{\,5}\, u_2^{\,5}$
$- 3080\, t\, u_2^{\,5}\, u_3^{\,4}\, u_4^{\,2} + 2980\, t\, u_2^{\,6}\, u_3^{\,2}\, u_4^{\,3} + 162\, t^3\, u_5\, u_3^{\,7}\, u_4 + 1710\, t\, u_2^{\,4}\, u_3^{\,6}\, u_4$
$- 45\, t^4\, u_5^{\,2}\, u_3^{\,4}\, u_4^{\,2} - 428\, t^3\, u_2^{\,2}\, u_3^{\,2}\, u_4^{\,5} + 15\, t^4\, u_2^{\,2}\, u_5^{\,2}\, u_4^{\,4} + 6360\, t^2\, u_2^{\,3}\, u_3^{\,4}\, u_4^{\,3}$
$- 4500\, t^2\, u_2^{\,2}\, u_3^{\,6}\, u_4^{\,2} - 4170\, t^2\, u_2^{\,4}\, u_3^{\,2}\, u_4^{\,4} - 396\, t^2\, u_5^{\,2}\, u_2^{\,6}\, u_4^{\,2} - 54\, t^3\, u_2\, u_5^{\,2}\, u_3^{\,6}$
$+ 360\, t^3\, u_2\, u_3^{\,4}\, u_4^{\,4} + 1620\, t^2\, u_2\, u_3^{\,8}\, u_4 + 162\, t^2\, u_2^{\,2}\, u_5\, u_3^{\,7} + 250\, t^3\, u_2^{\,4}\, u_5^{\,2}\, u_4^{\,3}$
$- 3\, t^5\, u_3^{\,2}\, u_5^{\,4}\, u_4 - 55\, t^2\, u_5^{\,2}\, u_3^{\,4}\, u_2^{\,4} + 40\, t^3\, u_2^{\,3}\, u_5^{\,3}\, u_3^{\,3} + 162\, t\, u_3^{\,2}\, u_5^{\,2}\, u_2^{\,7}$
$- 648\, t\, u_2^{\,8}\, u_5^{\,2}\, u_4 + 5\, t^5\, u_3\, u_5^{\,3}\, u_4^{\,3} - 8\, t^4\, u_2^{\,2}\, u_5^{\,4}\, u_3^{\,2} + 30\, t^4\, u_3^{\,3}\, u_5\, u_4^{\,4}$
$- 54\, t^2\, u_3\, u_2^{\,6}\, u_5^{\,3} + 810\, u_2^{\,9}\, u_3\, u_5\, u_4 + 12\, t^4\, u_2^{\,3}\, u_5^{\,4}\, u_4 - 40\, t^4\, u_3\, u_2^{\,2}\, u_5^{\,3}\, u_4^{\,2}$
$- 135\, t^3\, u_3\, u_2^{\,4}\, u_5^{\,3}\, u_4 - 666\, t^3\, u_2\, u_5\, u_3^{\,5}\, u_4^{\,2} - 400\, u_2^{\,6}\, u_3^{\,6} - 243\, u_5^{\,2}\, u_2^{\,10}$

$f_{18} := 1260\, t^3\, u_5^{\,2}\, u_3^{\,5}\, u_2^{\,6}\, u_4^{\,2} + 42330\, t^3\, u_3^{\,4}\, u_4^{\,4}\, u_2^{\,6}\, u_5 + 90\, t^6\, u_5^{\,5}\, u_3^{\,2}\, u_4^{\,2}\, u_2^{\,3}$
$- 1350\, u_4^{\,3}\, u_2^{\,14}\, u_5 + 135\, t^4\, u_2\, u_3^{\,10}\, u_4^{\,2}\, u_5 - 140\, t^3\, u_5^{\,3}\, u_3^{\,2}\, u_2^{\,8}\, u_4^{\,2}$
$- 510\, t^4\, u_2^{\,2}\, u_3^{\,8}\, u_4^{\,3}\, u_5 + t^7\, u_4^{\,10}\, u_5 + t^6\, u_2^{\,5}\, u_5^{\,7} - 45\, t^5\, u_3^{\,7}\, u_4^{\,6} - 800\, u_2^{\,11}\, u_3^{\,6}\, u_5$
$- 135\, t^4\, u_3^{\,11}\, u_4^{\,3} - 1620\, u_5^{\,2}\, u_2^{\,13}\, u_3^{\,3} - 5\, t^6\, u_4^{\,9}\, u_3^{\,3} + 800\, t\, u_2^{\,6}\, u_3^{\,11} + 2250\, u_4^{\,4}\, u_2^{\,13}\, u_3$
$+ 1620\, t^2\, u_3^{\,13}\, u_2^{\,3} + 81\, t^5\, u_5^{\,3}\, u_3^{\,10} - 2125\, u_2^{\,12}\, u_3^{\,3}\, u_4^{\,3} + 500\, u_2^{\,11}\, u_3^{\,5}\, u_4^{\,2} - t^7\, u_5^{\,6}\, u_3^{\,5}$
$- 22600\, t^3\, u_3^{\,2}\, u_4^{\,5}\, u_2^{\,7}\, u_5 + 940\, t^4\, u_2^{\,3}\, u_3^{\,6}\, u_4^{\,4}\, u_5 - 615\, t^5\, u_5^{\,3}\, u_3^{\,2}\, u_2^{\,4}\, u_4^{\,4}$
$+ 30265\, t^3\, u_3^{\,8}\, u_4^{\,2}\, u_5\, u_2^{\,4} + 180\, t^6\, u_2^{\,2}\, u_5^{\,3}\, u_3^{\,2}\, u_4^{\,5} - 2235\, t^5\, u_2^{\,2}\, u_5^{\,2}\, u_3^{\,5}\, u_4^{\,4}$
$- 980\, t^3\, u_5^{\,3}\, u_2^{\,9}\, u_4^{\,3} - 196\, t^5\, u_2^{\,5}\, u_5^{\,3}\, u_4^{\,5} - 73828\, t^3\, u_3^{\,5}\, u_4^{\,5}\, u_2^{\,5} - 14895\, t^2\, u_3^{\,11}\, u_4\, u_2^{\,4}$
$- 515\, t^3\, u_5^{\,4}\, u_2^{\,8}\, u_3^{\,3} + 1110\, t^4\, u_4^{\,8}\, u_2^{\,5}\, u_3 - 81\, t^3\, u_5^{\,5}\, u_2^{\,10} + 980\, t^5\, u_2^{\,3}\, u_3^{\,3}\, u_4^{\,5}\, u_5^{\,2}$
$+ 5\, t^7\, u_5^{\,5}\, u_3^{\,4}\, u_4^{\,2} + 105\, t^2\, u_5^{\,3}\, u_2^{\,9}\, u_3^{\,4} + 3645\, u_5^{\,2}\, u_2^{\,14}\, u_3\, u_4 + 210\, t^5\, u_2\, u_4^{\,7}\, u_3^{\,5}$
$+ 3600\, u_5\, u_2^{\,12}\, u_3^{\,4}\, u_4 - 3375\, u_5\, u_2^{\,13}\, u_3^{\,2}\, u_4^{\,2} - 7240\, t^3\, u_4^{\,7}\, u_2^{\,7}\, u_3 + 15\, t^6\, u_2\, u_5^{\,5}\, u_3^{\,6}$
$+ 30\, t^6\, u_3^{\,4}\, u_4^{\,7}\, u_5 + 10\, t^7\, u_5^{\,3}\, u_3^{\,2}\, u_4^{\,6} - 7290\, t^3\, u_2\, u_3^{\,13}\, u_4 + 225\, t^5\, u_3^{\,8}\, u_4^{\,4}\, u_5$
$+ 990\, t^4\, u_2\, u_3^{\,9}\, u_4^{\,4} + 5445\, t\, u_2^{\,12}\, u_4^{\,4}\, u_5 - 8256\, t^2\, u_2^{\,10}\, u_4^{\,5}\, u_5 - 81\, t^6\, u_3^{\,5}\, u_4^{\,5}\, u_5^{\,2}$
$+ 34340\, t^3\, u_3^{\,3}\, u_4^{\,6}\, u_2^{\,6} - 270\, t^5\, u_5^{\,2}\, u_3^{\,9}\, u_4^{\,2} - 45\, t^5\, u_4^{\,8}\, u_2^{\,4}\, u_5 - 8700\, t\, u_2^{\,11}\, u_4^{\,5}\, u_3$
$- 37950\, t\, u_2^{\,9}\, u_3^{\,5}\, u_4^{\,3} + 31150\, t\, u_2^{\,10}\, u_3^{\,3}\, u_4^{\,4} - 45\, t^6\, u_5^{\,4}\, u_3^{\,7}\, u_4 + 22275\, t\, u_2^{\,8}\, u_3^{\,7}\, u_4^{\,2}$
$- 2481\, t\, u_5^{\,2}\, u_2^{\,10}\, u_3^{\,5} - 6600\, t\, u_2^{\,7}\, u_3^{\,9}\, u_4 - 1215\, t\, u_5^{\,3}\, u_2^{\,12}\, u_3^{\,2} + 100\, t^6\, u_5^{\,3}\, u_3^{\,6}\, u_4^{\,3}$
$+ 10\, t^6\, u_3\, u_2\, u_4^{\,10} - 5\, t^7\, u_3\, u_4^{\,8}\, u_5^{\,2} + 56110\, t^2\, u_2^{\,5}\, u_3^{\,9}\, u_4^{\,2} + 5575\, t^3\, u_4^{\,6}\, u_2^{\,8}\, u_5$
$- 5\, t^5\, u_2^{\,6}\, u_5^{\,5}\, u_4^{\,2} - 109660\, t^2\, u_2^{\,6}\, u_3^{\,7}\, u_4^{\,3} - 59835\, t^2\, u_2^{\,8}\, u_3^{\,3}\, u_4^{\,5} - 180\, t^4\, u_5^{\,5}\, u_2^{\,8}\, u_4$
$- 60\, t^5\, u_2^{\,2}\, u_5^{\,4}\, u_3^{\,7} - 3155\, t^4\, u_3^{\,3}\, u_4^{\,7}\, u_2^{\,4} - 2940\, t^4\, u_2^{\,2}\, u_3^{\,7}\, u_4^{\,5} + 30510\, t^3\, u_2^{\,2}\, u_3^{\,11}\, u_4^{\,2}$
$+ 1215\, t^3\, u_2^{\,2}\, u_3^{\,12}\, u_5 - 1390\, t^4\, u_4^{\,7}\, u_2^{\,6}\, u_5 + 20\, t^6\, u_2^{\,2}\, u_4^{\,9}\, u_5 + 92290\, t^3\, u_2^{\,4}\, u_3^{\,7}\, u_4^{\,4}$
$- 15\, t^5\, u_2^{\,6}\, u_5^{\,6}\, u_3 + 515\, t^4\, u_2^{\,3}\, u_5^{\,3}\, u_3^{\,8} + 12310\, t^2\, u_2^{\,9}\, u_4^{\,6}\, u_3 - 370\, t^5\, u_2^{\,2}\, u_4^{\,8}\, u_3^{\,3}$
$- 69220\, t^3\, u_2^{\,3}\, u_3^{\,9}\, u_4^{\,3} + 260\, t^5\, u_4^{\,9}\, u_2^{\,3}\, u_3 + 4300\, t^4\, u_2^{\,3}\, u_3^{\,5}\, u_4^{\,6} - 105\, t^3\, u_5^{\,2}\, u_3^{\,9}\, u_2^{\,4} +$

$$
\begin{aligned}
&+ 1890\,t^2\,{u_5}^3\,{u_2}^{11}\,{u_4}^2 - 25\,t^4\,{u_2}^7\,{u_5}^3\,{u_4}^4 + 114960\,t^2\,{u_2}^7\,{u_3}^5\,{u_4}^4 + 2481\,t^2\,{u_2}^5\,{u_3}^{10}\,u_5\\
&- 10\,t^7\,{u_5}^4\,{u_3}^3\,{u_4}^4 + 40\,t^6\,{u_5}^3\,{u_4}^6\,{u_2}^3 + 60\,t^4\,{u_3}^2\,{u_5}^5\,{u_2}^7 + 10\,t^6\,{u_5}^5\,{u_4}^3\,{u_2}^4 +\\
&- 3900\,t^4\,{u_3}^5\,{u_4}^3\,{u_5}^2\,{u_2}^4 - 265\,t^6\,u_2\,{u_3}^4\,{u_5}^3\,{u_4}^4 - 1240\,t^4\,{u_3}^4\,{u_4}^5\,{u_2}^4\,u_5\\
&+ 10595\,t\,u_5\,{u_2}^{10}\,{u_3}^4\,{u_4}^2 + 30\,t^5\,{u_5}^5\,{u_3}^2\,{u_2}^5\,u_4 - 17520\,t\,u_5\,{u_2}^{11}\,{u_3}^2\,{u_4}^3\\
&+ 270\,t\,{u_5}^2\,{u_2}^{12}\,u_3\,{u_4}^2 - 1650\,t\,{u_2}^9\,{u_3}^6\,u_5\,u_4 + 43605\,t^2\,u_5\,{u_2}^9\,{u_3}^2\,{u_4}^4\\
&- 360\,t^5\,u_2\,{u_3}^8\,{u_5}^3\,u_4 + 1320\,t^5\,u_2\,{u_3}^7\,{u_5}^2\,{u_4}^3 - 1110\,t^5\,u_2\,{u_4}^5\,{u_3}^6\,u_5\\
&+ 110\,t^6\,u_2\,{u_3}^5\,{u_5}^4\,{u_4}^2 - 48360\,t^3\,{u_2}^5\,{u_3}^6\,{u_4}^3\,u_5 - 7590\,t^2\,{u_5}^2\,{u_2}^{10}\,u_3\,{u_4}^3\\
&+ 2920\,t^4\,{u_2}^3\,{u_5}^2\,{u_3}^7\,{u_4}^2 - 65\,t^6\,u_2\,{u_4}^8\,{u_3}^2\,u_5 + 1275\,t^4\,{u_3}^2\,{u_4}^6\,{u_2}^5\,u_5\\
&+ 5580\,t\,{u_5}^2\,{u_2}^{11}\,{u_3}^3\,u_4 + 990\,t^3\,{u_5}^4\,{u_2}^9\,u_3\,u_4 + 270\,t^5\,u_3\,{u_4}^3\,{u_2}^5\,{u_5}^4\\
&+ 14360\,t^2\,{u_2}^9\,{u_3}^3\,{u_5}^2\,{u_4}^2 + 57060\,t^2\,{u_2}^7\,{u_3}^6\,u_5\,{u_4}^2 - 15\,t^6\,u_3\,{u_5}^6\,{u_2}^4\,u_4\\
&- 30\,t^6\,{u_2}^2\,{u_5}^4\,{u_3}^3\,{u_4}^3 + 1995\,t^5\,{u_2}^2\,{u_3}^4\,{u_4}^6\,u_5 + 800\,t^3\,{u_5}^2\,{u_3}^3\,{u_2}^7\,{u_4}^3\\
&- 4755\,t^2\,{u_2}^8\,{u_3}^5\,{u_5}^2\,u_4 + 220\,t^5\,{u_2}^3\,{u_5}^3\,{u_3}^4\,{u_4}^3 - 9540\,t^3\,{u_2}^3\,{u_3}^{10}\,u_4\,u_5\\
&- 5760\,t^4\,{u_3}^2\,{u_4}^3\,{u_2}^6\,{u_5}^3 - 120\,t^6\,{u_2}^2\,u_3\,{u_4}^7\,{u_5}^2 - 780\,t^4\,{u_5}^4\,{u_3}^3\,{u_2}^6\,u_4\\
&+ 195\,t^3\,u_3\,{u_4}^4\,{u_2}^8\,{u_5}^2 - 19020\,t^2\,{u_2}^6\,{u_3}^8\,u_5\,u_4 - 77790\,t^2\,{u_2}^8\,{u_3}^4\,u_5\,{u_4}^3\\
&- 2700\,t^2\,{u_5}^3\,{u_2}^{10}\,{u_3}^2\,u_4 - 480\,t^3\,{u_5}^2\,{u_3}^7\,{u_2}^5\,u_4 + 500\,t^5\,{u_2}^2\,{u_5}^3\,{u_3}^6\,{u_4}^2\\
&- 60\,t^6\,{u_2}^2\,{u_5}^5\,{u_3}^4\,u_4 - 120\,t^6\,u_3\,{u_5}^4\,{u_2}^3\,{u_4}^4 + 1420\,t^4\,{u_5}^4\,{u_2}^7\,u_3\,{u_4}^2\\
&- 675\,t^4\,{u_2}^2\,{u_3}^9\,u_4\,{u_5}^2 + 2945\,t^4\,u_3\,{u_4}^5\,{u_2}^6\,{u_5}^2 + 660\,t^5\,u_3\,{u_4}^6\,{u_2}^4\,{u_5}^2\\
&- 1320\,t^5\,{u_2}^3\,{u_3}^2\,{u_4}^7\,u_5 + 120\,t^5\,{u_2}^3\,{u_5}^4\,{u_3}^5\,u_4 - 225\,t^5\,{u_5}^4\,{u_3}^3\,{u_2}^4\,{u_4}^2 + 729\,t^3\,{u_3}^{15}\\
&+ 180\,t^4\,{u_2}^5\,{u_4}^4\,{u_3}^3\,{u_5}^2 + 7020\,t^4\,{u_5}^3\,{u_3}^4\,{u_2}^5\,{u_4}^2 + 200\,t^6\,u_2\,{u_4}^6\,{u_3}^3\,{u_5}^2\\
&- 3120\,t^4\,{u_5}^3\,{u_3}^6\,{u_2}^4\,u_4 - 120\,t^3\,{u_5}^3\,{u_3}^4\,{u_2}^7\,u_4 - 729\,{u_5}^3\,{u_2}^{15}.
\end{aligned}
$$

The generating invariants $f_2, f_4, f_6, f_{10}, f_{15}$. in the case $n=6$.

$$f_2 := -t\,u_6 - 15\,u_2\,u_4 + 10\,u_3{}^2.$$

$$f_4 := 1200\,u_2\,u_3{}^2\,u_4 + 300\,t\,u_5{}^2\,u_2 + 300\,u_6\,u_2{}^3 + 320\,t\,u_6\,u_3{}^2 - 600\,t\,u_3\,u_5\,u_4 - \\ - 330\,t\,u_2\,u_6\,u_4 - 400\,u_3{}^4 - t^2\,u_6{}^2 + 300\,t\,u_4{}^3 - 525\,u_2{}^2\,u_4{}^2 - 600\,u_2{}^2\,u_3\,u_5$$

$$f_6 = 12\,t^2\,u_3\,u_5{}^3 + 16\,t\,u_3{}^2\,u_4{}^3 + 8\,t\,u_2{}^3\,u_6{}^2 + 36\,u_2{}^3\,u_4{}^3 - 81\,u_5{}^2\,u_2{}^4 - 24\,t\,u_2\,u_4{}^4 \\ - 337\,u_2{}^2\,u_3{}^2\,u_4{}^2 + 16\,u_6\,u_2{}^3\,u_3{}^2 + 318\,u_2{}^3\,u_3\,u_5\,u_4 - 152\,u_2{}^2\,u_3{}^3\,u_5 - 24\,u_2{}^4\,u_6\,u_4 \\ - 16\,t\,u_6\,u_3{}^4 + 288\,u_2\,u_3{}^4\,u_4 - 10\,t^2\,u_3\,u_5\,u_4\,u_6 - 46\,t\,u_2{}^2\,u_3\,u_5\,u_6 + 30\,t\,u_2{}^2\,u_4\,u_5{}^2 \\ - 64\,u_3{}^6 + 8\,t^2\,u_6\,u_4{}^3 - 6\,t\,u_2\,u_3\,u_5\,u_4{}^2 - t^2\,u_3{}^2\,u_6{}^2 - 9\,t^2\,u_5{}^2\,u_4{}^2 - 4\,t^2\,u_5{}^2\,u_2\,u_6 \\ + 50\,t\,u_2\,u_3{}^2\,u_6\,u_4 + 4\,t\,u_2\,u_3{}^2\,u_5{}^2 + 4\,t^2\,u_2\,u_4\,u_6{}^2 - 8\,t\,u_3{}^3\,u_5\,u_4 - 8\,t\,u_2{}^2\,u_6\,u_4{}^2.$$

$$f_{10} = -960\,t\,u_2{}^2\,u_4{}^5\,u_3{}^2 - 126\,t\,u_4{}^4\,u_2{}^4\,u_6 + 900\,t\,u_2\,u_4{}^4\,u_3{}^4 - 510\,t^2\,u_3{}^4\,u_5{}^2\,u_4{}^2 + \\ + 12\,t^2\,u_4{}^3\,u_3{}^4\,u_6 + 243\,t\,u_2{}^5\,u_5{}^4 - 84\,t^2\,u_3{}^5\,u_5\,u_4\,u_6 - 96\,t\,u_2{}^4\,u_6{}^2\,u_3{}^2\,u_4 - \\ - 24\,t^3\,u_3{}^2\,u_4\,u_5{}^4 - 468\,t^2\,u_2\,u_3\,u_4{}^5\,u_5 + 12\,t^3\,u_3\,u_5\,u_2{}^2\,u_6{}^3 + 420\,t^2\,u_2{}^2\,u_3{}^2\,u_5{}^4 \\ + 2016\,t\,u_2{}^4\,u_6\,u_3\,u_5\,u_4{}^2 - 204\,t\,u_2\,u_3{}^5\,u_5\,u_4{}^2 - 12\,t^3\,u_3{}^3\,u_4\,u_6{}^2\,u_5 + 93\,t\,u_2{}^5\,u_6{}^2\,u_4{}^2 \\ - 21\,t^2\,u_4\,u_2{}^4\,u_6{}^3 + 36\,u_2{}^7\,u_6{}^2\,u_4 - 87\,t^2\,u_2\,u_6{}^2\,u_3{}^4\,u_4 - 1176\,t\,u_2\,u_6\,u_3{}^6\,u_4 \\ + 225\,t^2\,u_2{}^2\,u_5{}^2\,u_4{}^4 + 6\,t\,u_3{}^2\,u_5{}^2\,u_2{}^4\,u_6 + t^4\,u_5{}^6 - 48\,t^3\,u_2\,u_3{}^2\,u_6{}^2\,u_5{}^2 - 96\,t^2\,u_2\,u_3{}^2\,u_4{}^4\,u_6 \\ - 1260\,t\,u_2{}^3\,u_3{}^2\,u_5{}^2\,u_4{}^2 - 114\,t^3\,u_3{}^2\,u_4{}^2\,u_6\,u_5{}^2 - 228\,t^2\,u_2\,u_3{}^4\,u_5{}^2\,u_6 \\ + 51\,t^2\,u_2{}^4\,u_6{}^2\,u_5{}^2 + 276\,t\,u_2{}^2\,u_6\,u_3{}^5\,u_5 + 720\,t\,u_2{}^2\,u_3{}^4\,u_5{}^2\,u_4 - 216\,t^2\,u_2{}^3\,u_6\,u_3\,u_5{}^3 \\ + 60\,t^2\,u_2{}^2\,u_6{}^2\,u_3{}^3\,u_5 + 1476\,u_3\,u_5\,u_2{}^6\,u_4\,u_6 + 93\,t^2\,u_2{}^2\,u_4{}^5\,u_6 + 180\,t\,u_3\,u_4\,u_5{}^3\,u_2{}^4 \\ + 12\,t^3\,u_2\,u_3\,u_5{}^5 - 84\,t^2\,u_3\,u_4\,u_2{}^3\,u_6{}^2\,u_5 - 576\,t\,u_4\,u_2{}^5\,u_6\,u_5{}^2 - 3840\,u_2\,u_3{}^8\,u_4 \\ + 6\,t^3\,u_2\,u_4{}^2\,u_5{}^4 - 3\,t^4\,u_4\,u_6\,u_5{}^4 + 84\,t^3\,u_3\,u_6\,u_4{}^4\,u_5 - 11405\,u_2{}^3\,u_4{}^3\,u_3{}^4 \\ + 192\,t\,u_6\,u_3{}^8 + 360\,t^2\,u_3{}^5\,u_5{}^3 + t^3\,u_3{}^4\,u_6{}^3 - 6\,t^3\,u_5{}^2\,u_4{}^5 + 18\,t^3\,u_4\,u_2{}^2\,u_6{}^2\,u_5{}^2 \\ + 36\,t^2\,u_2\,u_4{}^7 - 24\,t^2\,u_4{}^6\,u_3{}^2 - 20\,t^3\,u_4{}^6\,u_6 - t^3\,u_2{}^3\,u_6{}^4 - 20\,t\,u_2{}^6\,u_6{}^3 \\ + 24\,t^2\,u_2{}^2\,u_3{}^2\,u_6{}^2\,u_4{}^2 + 3960\,u_4{}^4\,u_2{}^4\,u_3{}^2 - 9\,t^3\,u_2{}^2\,u_6\,u_5{}^4 + 3\,t^4\,u_4{}^2\,u_6{}^2\,u_5{}^2 \\ + 78\,t^2\,u_2{}^3\,u_6{}^2\,u_4{}^3 + 396\,t^2\,u_4\,u_2{}^2\,u_6\,u_3{}^2\,u_5{}^2 + 162\,t^2\,u_4{}^2\,u_2{}^3\,u_6\,u_5{}^2 \\ + 1050\,t^2\,u_2\,u_3{}^2\,u_5{}^2\,u_4{}^3 + 60\,t\,u_2{}^5\,u_6{}^2\,u_3\,u_5 - 660\,t^2\,u_2{}^2\,u_3\,u_6\,u_4{}^3\,u_5 \\ + 2499\,t\,u_2{}^2\,u_6\,u_4{}^2\,u_3{}^4 - 840\,t^2\,u_2\,u_4\,u_3{}^3\,u_5{}^3 + 552\,t^2\,u_2\,u_3{}^3\,u_5\,u_4{}^2\,u_6 \\ - 24\,t^3\,u_3\,u_4\,u_2\,u_6\,u_5{}^3 + 512\,u_3{}^{10} + 84\,t^3\,u_2\,u_3\,u_4{}^2\,u_6{}^2\,u_5 - 420\,t^2\,u_2{}^2\,u_4{}^2\,u_3\,u_5{}^3 \\ - 5400\,u_4{}^3\,u_2{}^5\,u_3\,u_5 - 624\,u_2{}^5\,u_6\,u_3{}^3\,u_5 - 960\,u_2{}^5\,u_6\,u_3{}^2\,u_4{}^2 - 9780\,u_2{}^3\,u_3{}^5\,u_5\,u_4 \\ - 12\,t^3\,u_2\,u_4{}^3\,u_6\,u_5{}^2 + 12\,t\,u_2{}^3\,u_6{}^2\,u_3{}^4 + 20\,t^3\,u_3\,u_4{}^3\,u_5{}^3 + 96\,t\,u_3{}^7\,u_5\,u_4 \\ + 2880\,t\,u_2{}^3\,u_4{}^4\,u_3\,u_5 - 1368\,t\,u_2{}^3\,u_3{}^3\,u_5\,u_4\,u_6 + 72\,t^3\,u_6\,u_3{}^3\,u_5{}^3 + 240\,t^2\,u_4{}^4\,u_3{}^3\,u_5 \\ + 162\,u_4{}^5\,u_2{}^5 - 1824\,t\,u_2{}^3\,u_4{}^3\,u_3{}^2\,u_6 - 1260\,t\,u_2{}^2\,u_3{}^3\,u_5\,u_4{}^3 - 486\,u_2{}^7\,u_6\,u_5{}^2 \\ - 21\,t^3\,u_2\,u_6{}^2\,u_4{}^4 + 2370\,u_3{}^4\,u_5{}^2\,u_2{}^4 - 24\,u_2{}^6\,u_6{}^2\,u_3{}^2 - 135\,u_4{}^3\,u_2{}^6\,u_6 \\ + 10320\,u_2{}^2\,u_4{}^2\,u_3{}^6 - 200\,u_2{}^3\,u_6\,u_3{}^6 + 160\,t\,u_2{}^3\,u_3{}^3\,u_5{}^3 - 48\,t\,u_2\,u_3{}^6\,u_5{}^2 \\ + 14700\,u_3{}^3\,u_5\,u_2{}^4\,u_4{}^2 - 6174\,u_3{}^2\,u_5{}^2\,u_2{}^5\,u_4 + 900\,u_2{}^4\,u_6\,u_3{}^4\,u_4 - 45\,t^2\,u_4\,u_5{}^4\,u_2{}^3 \\ - 1080\,t\,u_4{}^3\,u_2{}^4\,u_5{}^2 - 18\,t^3\,u_2{}^2\,u_4{}^2\,u_6{}^3 + 1701\,u_4{}^2\,u_2{}^6\,u_5{}^2 + 972\,u_3\,u_5{}^3\,u_2{}^6 \\ + 24\,t^2\,u_6{}^2\,u_3{}^6 - t^4\,u_4{}^3\,u_6{}^3 - 135\,t\,u_2{}^3\,u_4{}^6 - 200\,t\,u_4{}^3\,u_3{}^6 + 1920\,u_2{}^2\,u_3{}^7\,u_5$$

$$
\begin{aligned}
f_{15} = {}& 432\,t^4\,u_4{}^{10}\,u_5 - 2816\,t^4\,u_5{}^6\,u_3{}^5 - 8000\,u_2{}^6\,u_3{}^7\,u_6{}^2 + 8000\,t^2\,u_4{}^6\,u_3{}^7 - 18225\,u_2{}^9\,u_4{}^3\,u_5{}^3 \\
& - 756\,t^3\,u_5{}^7\,u_2{}^5 - 12800\,u_2{}^6\,u_3{}^6\,u_5{}^3 - 19683\,u_2{}^{10}\,u_5{}^5 + 1600\,u_2{}^9\,u_6{}^3\,u_3{}^3 \\
& - 1600\,t^3\,u_4{}^9\,u_3{}^3 + 36\,t^5\,u_4{}^6\,u_5{}^3\,u_6 - 31860\,u_2{}^8\,u_3{}^3\,u_5{}^4 - 32768\,t^2\,u_3{}^{10}\,u_5{}^3 \\
& + 9\,t^5\,u_4{}^5\,u_5{}^5 + 2160\,u_2{}^{11}\,u_6{}^3\,u_5 - 5670\,t^4\,u_2{}^2\,u_3\,u_4{}^4\,u_6{}^2\,u_5{}^2 - 2960\,t\,u_2{}^6\,u_6{}^3\,u_3{}^5 \\
& + 105300\,t^2\,u_2{}^5\,u_4\,u_3{}^2\,u_5{}^5 + 675\,t^3\,u_2{}^4\,u_4{}^3\,u_5{}^5 + 2960\,t^3\,u_4{}^6\,u_3{}^5\,u_6 \\
& - 132\,t^5\,u_2\,u_4{}^5\,u_6{}^3\,u_5 - 27000\,t^2\,u_2{}^3\,u_4{}^9\,u_3 - 6960\,t^4\,u_2\,u_3{}^3\,u_4{}^3\,u_6{}^2\,u_5{}^2 \\
& + 15\,t^4\,u_5\,u_2{}^5\,u_6{}^5 - 945\,t^3\,u_2{}^3\,u_4{}^6\,u_5{}^3 - 45\,t^4\,u_2{}^3\,u_5{}^7\,u_4 - 50625\,u_2{}^8\,u_4{}^5\,u_6\,u_3 \\
& + 22536\,t^2\,u_4\,u_3{}^3\,u_5{}^2\,u_2{}^5\,u_6{}^2 - 3\,t^5\,u_2{}^2\,u_5{}^7\,u_6 + 464\,t^4\,u_4{}^6\,u_6{}^2\,u_3{}^3 \\
& + 11340\,t\,u_4{}^3\,u_2{}^7\,u_5{}^2\,u_3\,u_6 + 22275\,t^2\,u_4{}^8\,u_2{}^4\,u_5 + 70200\,u_2{}^8\,u_6\,u_5{}^2\,u_3{}^3\,u_4 \\
& - 5760\,t^3\,u_2{}^2\,u_3{}^4\,u_5{}^3\,u_4{}^2\,u_6 + 61440\,t^2\,u_3{}^9\,u_5{}^2\,u_4{}^2 + 192\,t^3\,u_2{}^3\,u_3{}^5\,u_6{}^4 \\
& + 76545\,u_2{}^9\,u_3\,u_5{}^4\,u_4 + 24\,t^4\,u_3{}^3\,u_6{}^5\,u_2{}^3 + 144\,t\,u_2{}^9\,u_6{}^4\,u_3 - 3\,t^5\,u_4\,u_5\,u_2{}^3\,u_6{}^5 \\
& - t^6\,u_4{}^3\,u_6{}^5\,u_3 + 8667\,t^2\,u_2{}^7\,u_6\,u_5{}^5 - 225\,t^4\,u_2{}^2\,u_4{}^4\,u_5{}^5 - 76800\,t^2\,u_2{}^4\,u_3{}^4\,u_5{}^5 \\
& - 129\,t^4\,u_2{}^4\,u_6{}^2\,u_5{}^5 + 5418\,t^2\,u_2{}^6\,u_4{}^2\,u_5{}^2\,u_3\,u_6{}^2 - 11136\,t^3\,u_4{}^5\,u_3{}^5\,u_5{}^2 \\
& - 5760\,t^3\,u_2{}^3\,u_5{}^6\,u_3{}^3 - 360\,t^2\,u_5\,u_2{}^8\,u_6{}^4 + 7800\,t^4\,u_3{}^2\,u_4{}^6\,u_5{}^3 - 24\,t^3\,u_2{}^6\,u_6{}^5\,u_3 \\
& + 512\,t^2\,u_2{}^3\,u_3{}^7\,u_6{}^3 - 20250\,t^2\,u_2{}^6\,u_3\,u_5{}^6 - 5400\,t^3\,u_2{}^2\,u_4{}^9\,u_5 - 24\,t^5\,u_4{}^3\,u_6{}^4\,u_3{}^3 \\
& + 5670\,t^2\,u_2{}^6\,u_4{}^2\,u_5{}^5 + 66000\,t^2\,u_2{}^2\,u_3{}^3\,u_4{}^8 - 2940\,t^4\,u_2{}^2\,u_3{}^3\,u_6{}^2\,u_5{}^4 \\
& + 137\,t^3\,u_2{}^6\,u_6{}^3\,u_5{}^3 + 7440\,t^3\,u_3{}^4\,u_4{}^7\,u_5 - 38400\,t^2\,u_4{}^4\,u_3{}^8\,u_5 + 24\,t^5\,u_3\,u_4{}^6\,u_6{}^3 \\
& - 192\,t^4\,u_4{}^3\,u_6{}^3\,u_3{}^5 - 512\,t^3\,u_4{}^3\,u_3{}^7\,u_6{}^2 + 2\,t^6\,u_5{}^9 - 12288\,t^3\,u_3{}^8\,u_5{}^3\,u_6 \\
& + 18549\,t^2\,u_2{}^5\,u_4{}^5\,u_5{}^3 + 5120\,t^3\,u_4{}^3\,u_3{}^6\,u_5{}^3 - 464\,t^2\,u_3{}^3\,u_2{}^6\,u_6{}^4 \\
& - 92160\,t^2\,u_2{}^2\,u_3{}^7\,u_5{}^4 + 29340\,t^3\,u_2{}^2\,u_3\,u_4{}^7\,u_5{}^2 - 180\,t^4\,u_2\,u_4{}^7\,u_5{}^3 \\
& - 128\,t^5\,u_5{}^6\,u_3{}^3\,u_6 + 163215\,t\,u_4{}^2\,u_2{}^7\,u_5{}^4\,u_3 - 129024\,t^2\,u_2{}^2\,u_3{}^7\,u_4\,u_6\,u_5{}^2 \\
& + 45\,t^5\,u_2\,u_4{}^2\,u_5{}^7 - 24300\,t\,u_4{}^3\,u_6{}^2\,u_2{}^8\,u_5 - 1536\,t^4\,u_3{}^6\,u_5{}^3\,u_6{}^2 \\
& - 101376\,t\,u_2{}^5\,u_3{}^5\,u_5{}^4 - 11100\,t^4\,u_3{}^3\,u_4{}^4\,u_5{}^4 - 72\,t^5\,u_4{}^7\,u_6{}^2\,u_5 \\
& + 13527\,t\,u_4{}^2\,u_2{}^8\,u_5{}^3\,u_6 - 1200\,t^3\,u_2{}^4\,u_3{}^3\,u_6{}^4\,u_4 + 2781\,t^3\,u_2{}^5\,u_6\,u_5{}^5\,u_4 \\
& + 26568\,t\,u_2{}^8\,u_6{}^2\,u_3\,u_4\,u_5{}^2 - 30\,t^5\,u_3\,u_4{}^3\,u_5{}^6 + t^5\,u_3\,u_2{}^3\,u_6{}^6 + 10000\,t\,u_2{}^3\,u_4{}^6\,u_3{}^5 \\
& + 2\,t^5\,u_5{}^3\,u_6{}^4\,u_2{}^3 + 3972\,t^4\,u_2\,u_3{}^2\,u_4{}^5\,u_6{}^2\,u_5 - 168960\,t^2\,u_2{}^2\,u_3{}^6\,u_4{}^2\,u_5{}^3 \\
& - 52800\,t^2\,u_2{}^2\,u_3{}^5\,u_4{}^5\,u_6 + 40152\,t^2\,u_2{}^5\,u_6\,u_5{}^4\,u_3{}^3 + 33792\,t^2\,u_3{}^6\,u_4\,u_6{}^2\,u_2{}^3\,u_5 \\
& + 3456\,t^4\,u_4{}^2\,u_6{}^2\,u_3{}^5\,u_5{}^2 + 81360\,t^3\,u_2{}^2\,u_3{}^3\,u_4{}^4\,u_6\,u_5{}^2 + 588\,t^4\,u_3{}^2\,u_4\,u_5\,u_6{}^4\,u_2{}^3 \\
& - 337920\,t^2\,u_2\,u_3{}^7\,u_5{}^2\,u_4{}^3 - 2400\,t\,u_2{}^6\,u_6{}^2\,u_3{}^4\,u_4\,u_5 - 272400\,t^2\,u_2{}^3\,u_4{}^4\,u_3{}^4\,u_6\,u_5 \\
& - 8808\,t^4\,u_2\,u_3\,u_4{}^6\,u_5{}^2\,u_6 - 21870\,t\,u_4\,u_2{}^8\,u_5{}^5 + 1092\,t^2\,u_2{}^7\,u_6{}^3\,u_3\,u_5{}^2 \\
& + 99900\,t^2\,u_2{}^3\,u_3{}^2\,u_4{}^7\,u_5 - 288\,t^4\,u_2{}^2\,u_5\,u_3{}^4\,u_6{}^4 + 9120\,t^3\,u_3{}^4\,u_4\,u_6{}^3\,u_2{}^3\,u_5 \\
& - 468720\,t\,u_3{}^2\,u_4{}^3\,u_2{}^6\,u_5{}^3 - 48000\,t\,u_2{}^3\,u_4{}^4\,u_3{}^6\,u_5 - 16200\,u_4{}^2\,u_2{}^{10}\,u_6{}^2\,u_5 \\
& + 15\,t^6\,u_4{}^2\,u_6{}^2\,u_5{}^5 + 50625\,t\,u_2{}^5\,u_4{}^8\,u_3 - 1965\,t^4\,u_2{}^2\,u_3\,u_4{}^3\,u_6\,u_5{}^4 \\
& + 49152\,t^3\,u_2\,u_3{}^6\,u_5{}^3\,u_4\,u_6 + 84\,t^5\,u_2\,u_3\,u_4{}^3\,u_6{}^3\,u_5{}^2 + 24000\,u_3{}^6\,u_5\,u_6\,u_2{}^6\,u_4 \\
& + 76800\,t\,u_2{}^3\,u_3{}^7\,u_5{}^2\,u_4{}^2 - 19968\,t^3\,u_2{}^2\,u_3{}^5\,u_6\,u_5{}^4 - 3504\,t\,u_2{}^8\,u_6{}^3\,u_3{}^2\,u_5 \\
& - 320000\,t^2\,u_2{}^3\,u_3{}^4\,u_4{}^3\,u_5{}^3 - 40960\,t\,u_2{}^3\,u_3{}^8\,u_5{}^3 - 6075\,t\,u_2{}^7\,u_4{}^5\,u_6\,u_5 \\
& - 7680\,t^3\,u_2\,u_3{}^5\,u_5{}^4\,u_4{}^2 + 18009\,t^2\,u_2{}^6\,u_6\,u_5{}^3\,u_4{}^3 + 132\,t^4\,u_4{}^2\,u_5\,u_2{}^4\,u_6{}^4 \\
& + 9216\,t^3\,u_2\,u_3{}^7\,u_6{}^2\,u_5{}^2 + 54999\,t\,u_2{}^8\,u_3\,u_5{}^4\,u_6 - 9936\,t^2\,u_2{}^4\,u_3{}^5\,u_6{}^2\,u_5{}^2 \\
& - 2880\,t\,u_3{}^6\,u_5\,u_2{}^5\,u_6{}^2 - 207144\,t\,u_4\,u_2{}^7\,u_5{}^3\,u_3{}^2\,u_6 - 12\,t^5\,u_5\,u_3{}^2\,u_2{}^2\,u_6{}^5 \\
& - 30720\,t^2\,u_2\,u_3{}^8\,u_5\,u_4{}^2\,u_6 - 49560\,t^3\,u_2{}^2\,u_3{}^2\,u_4{}^5\,u_5{}^3 + 1674\,t^3\,u_2{}^5\,u_4{}^2\,u_6{}^4\,u_3 \\
& + 534\,t^4\,u_2{}^3\,u_5{}^6\,u_3\,u_6 + 121500\,u_2{}^8\,u_4{}^3\,u_6\,u_3{}^2\,u_5 + 47925\,t\,u_2{}^7\,u_4{}^4\,u_3\,u_6{}^2 \\
& + 9600\,t^2\,u_2\,u_4{}^4\,u_6\,u_3{}^7 + 1200\,t^4\,u_2\,u_4{}^4\,u_6{}^3\,u_3{}^3 - 14136\,t\,u_2{}^7\,u_6{}^2\,u_3{}^3\,u_5{}^2 \\
& + 248400\,t^2\,u_4{}^4\,u_3{}^2\,u_5{}^3\,u_2{}^4 - 114000\,u_5\,u_3{}^4\,u_6\,u_2{}^7\,u_4{}^2 - 357600\,t\,u_3{}^5\,u_5{}^2\,u_2{}^4\,u_4{}^3 \\
& + 81456\,t\,u_2{}^6\,u_6\,u_3{}^4\,u_5{}^3 - 2880\,t^4\,u_4{}^3\,u_3{}^4\,u_5{}^3\,u_6 + 3\,t^6\,u_4{}^2\,u_6{}^4\,u_3\,u_5{}^2 \\
& + 41280\,t^3\,u_2{}^3\,u_4\,u_3{}^3\,u_5{}^4\,u_6 - 61764\,t^3\,u_2{}^2\,u_3{}^2\,u_4{}^6\,u_6\,u_5 - 33\,t^5\,u_2{}^2\,u_6{}^3\,u_5{}^4\,u_3 \\
& + 30375\,u_2{}^9\,u_4{}^4\,u_6\,u_5 + 1920\,t^4\,u_2\,u_3{}^3\,u_5{}^6\,u_4 + 8760\,t^4\,u_2{}^2\,u_4{}^2\,u_3{}^2\,u_6{}^2\,u_5{}^3 \\
& - 3240\,u_3{}^2\,u_2{}^9\,u_6\,u_5{}^3 + 57344\,t^2\,u_2{}^3\,u_3{}^6\,u_5{}^3\,u_6 - 66000\,u_4{}^2\,u_2{}^8\,u_6{}^2\,u_3{}^3 \\
& + 792\,t^4\,u_3{}^3\,u_4{}^5\,u_5{}^2\,u_6 + 131712\,t^2\,u_2{}^3\,u_3{}^5\,u_5{}^2\,u_4{}^2\,u_6 - 47925\,t^2\,u_2{}^4\,u_4{}^7\,u_6\,u_3
\end{aligned}
$$

$$+\,16186\,t^3\,u_2^{\,3}\,u_4^{\,6}\,u_6^{\,2}\,u_3+336\,t^5\,u_4^{\,2}\,u_3^{\,2}\,u_6\,u_5^{\,5}+8640\,t^4\,u_4^{\,2}\,u_3^{\,4}\,u_5^{\,5}$$
$$-\,30375\,t\,u_2^{\,6}\,u_4^{\,7}\,u_5+t^6\,u_5^{\,6}\,u_3\,u_6^{\,2}-615\,t^3\,u_2^{\,4}\,u_3\,u_5^{\,4}\,u_4^{\,2}\,u_6+30375\,u_2^{\,8}\,u_4^{\,4}\,u_5^{\,2}\,u_3$$
$$-\,48600\,t\,u_2^{\,6}\,u_6\,u_3^{\,2}\,u_4^{\,4}\,u_5-1800\,t^4\,u_2^{\,2}\,u_3^{\,3}\,u_4\,u_6^{\,3}\,u_5^{\,2}+69\,t^5\,u_2\,u_3\,u_4^{\,4}\,u_6^{\,4}$$
$$-\,61560\,t\,u_2^{\,7}\,u_3^{\,2}\,u_5^{\,5}-13932\,t\,u_2^{\,9}\,u_6^{\,2}\,u_5^{\,3}-42000\,t^2\,u_2\,u_4^{\,7}\,u_3^{\,5}$$
$$-\,20400\,u_2^{\,8}\,u_3^{\,4}\,u_6^{\,2}\,u_5-42525\,t\,u_2^{\,7}\,u_4^{\,4}\,u_5^{\,3}-11\,t^6\,u_4^{\,3}\,u_6^{\,3}\,u_5^{\,3}-64\,t^5\,u_3^{\,4}\,u_6^{\,3}\,u_5^{\,3}$$
$$+\,43740\,u_2^{\,10}\,u_6\,u_5^{\,3}\,u_4+6000\,u_2^{\,6}\,u_4^{\,2}\,u_3^{\,5}\,u_5^{\,2}-960\,t^4\,u_3^{\,2}\,u_2^{\,2}\,u_5^{\,7}$$
$$+\,61200\,u_3^{\,4}\,u_4\,u_5^{\,3}\,u_2^{\,7}-3600\,u_2^{\,10}\,u_6^{\,3}\,u_3\,u_4+42000\,u_2^{\,7}\,u_6^{\,2}\,u_3^{\,5}\,u_4$$
$$-\,12960\,u_2^{\,10}\,u_6^{\,2}\,u_3\,u_5^{\,2}-2400\,u_2^{\,7}\,u_6\,u_5^{\,2}\,u_3^{\,5}+27000\,u_2^{\,9}\,u_4^{\,3}\,u_6^{\,2}\,u_3$$
$$-\,7548\,t^2\,u_3^{\,2}\,u_4\,u_2^{\,6}\,u_6^{\,3}\,u_5+45000\,u_2^{\,7}\,u_4^{\,4}\,u_6\,u_3^{\,3}-10000\,u_4^{\,3}\,u_2^{\,6}\,u_6\,u_3^{\,5}$$
$$-\,144\,t^4\,u_3\,u_4^{\,9}\,u_6-9\,t^6\,u_4\,u_5^{\,7}\,u_6-64800\,u_4^{\,2}\,u_3^{\,2}\,u_5^{\,3}\,u_2^{\,8}-2880\,t^4\,u_3\,u_4^{\,8}\,u_5^{\,2}$$
$$+\,3600\,t^3\,u_2\,u_3\,u_4^{\,10}-4320\,t^2\,u_2^{\,6}\,u_3^{\,2}\,u_5^{\,3}\,u_6^{\,2}-255\,t^4\,u_2^{\,3}\,u_4\,u_6^{\,2}\,u_5^{\,4}\,u_3$$
$$+\,3\,t^6\,u_5\,u_4^{\,4}\,u_6^{\,4}+14280\,t^3\,u_2^{\,2}\,u_3\,u_4^{\,8}\,u_6+44400\,t\,u_2^{\,5}\,u_6\,u_3^{\,4}\,u_4^{\,3}\,u_5$$
$$+\,168960\,t^2\,u_2^{\,2}\,u_3^{\,6}\,u_4^{\,3}\,u_6\,u_5-122364\,t^2\,u_2^{\,5}\,u_4^{\,4}\,u_3\,u_6\,u_5^{\,2}-1128\,t^4\,u_2\,u_3\,u_4^{\,7}\,u_6^{\,2}$$
$$-\,1674\,t^4\,u_2^{\,2}\,u_3\,u_4^{\,5}\,u_6^{\,3}-4635\,t^3\,u_2^{\,4}\,u_4^{\,4}\,u_6\,u_5^{\,3}-8610\,t^3\,u_4^{\,5}\,u_6^{\,2}\,u_2^{\,4}\,u_5$$
$$-\,21480\,t^3\,u_2^{\,4}\,u_3^{\,2}\,u_5^{\,3}\,u_4\,u_6^{\,2}-3\,t^6\,u_4\,u_6^{\,3}\,u_5^{\,4}\,u_3-2820\,t^4\,u_2^{\,3}\,u_4^{\,2}\,u_6^{\,3}\,u_3\,u_5^{\,2}$$
$$+\,184320\,t^2\,u_2\,u_3^{\,8}\,u_5^{\,3}\,u_4-54480\,t^2\,u_2^{\,4}\,u_3^{\,4}\,u_4^{\,2}\,u_6^{\,2}\,u_5+4140\,t^3\,u_2^{\,5}\,u_6^{\,3}\,u_5^{\,2}\,u_3\,u_4$$
$$-\,130005\,t^2\,u_2^{\,4}\,u_4^{\,6}\,u_5^{\,2}\,u_3-4641\,t^3\,u_4^{\,2}\,u_2^{\,5}\,u_6^{\,2}\,u_5^{\,3}+35136\,t^2\,u_4^{\,2}\,u_3^{\,2}\,u_2^{\,5}\,u_6\,u_5^{\,3}$$
$$+\,36\,t^4\,u_2^{\,2}\,u_3^{\,2}\,u_6\,u_5^{\,5}\,u_4-88880\,t^3\,u_4^{\,3}\,u_2^{\,3}\,u_3^{\,2}\,u_5^{\,3}\,u_6-25488\,t^2\,u_2^{\,6}\,u_3\,u_5^{\,4}\,u_4\,u_6$$
$$+\,720\,t^3\,u_2^{\,4}\,u_3\,u_5^{\,6}\,u_4+93780\,t^2\,u_3^{\,2}\,u_2^{\,4}\,u_4^{\,5}\,u_6\,u_5+41160\,t^3\,u_2^{\,4}\,u_4^{\,3}\,u_6^{\,2}\,u_3\,u_5^{\,2}$$
$$+\,300\,t^5\,u_3\,u_4^{\,5}\,u_6^{\,2}\,u_5^{\,2}+4800\,t^3\,u_2\,u_4^{\,4}\,u_5^{\,3}\,u_3^{\,4}+753\,t^3\,u_2^{\,5}\,u_6^{\,2}\,u_5^{\,4}\,u_3$$
$$+\,27540\,t^2\,u_2^{\,5}\,u_4^{\,6}\,u_6\,u_5+7026\,t^2\,u_2^{\,7}\,u_6^{\,3}\,u_4^{\,2}\,u_5-153900\,u_2^{\,9}\,u_6\,u_5^{\,2}\,u_4^{\,2}\,u_3$$
$$+\,1128\,t^2\,u_2^{\,7}\,u_6^{\,4}\,u_3\,u_4+63216\,t^3\,u_2\,u_4^{\,5}\,u_6\,u_3^{\,4}\,u_5+9120\,t^3\,u_2^{\,3}\,u_4^{\,2}\,u_6^{\,2}\,u_3^{\,3}\,u_5^{\,2}$$
$$-\,6144\,t^2\,u_2^{\,2}\,u_3^{\,8}\,u_5\,u_6^{\,2}-14784\,t^3\,u_4^{\,4}\,u_3^{\,6}\,u_6\,u_5-9855\,t^2\,u_2^{\,7}\,u_6^{\,2}\,u_4\,u_5^{\,3}$$
$$+\,1536\,t^2\,u_2^{\,5}\,u_5\,u_3^{\,4}\,u_6^{\,3}-6384\,t^2\,u_2^{\,4}\,u_3^{\,5}\,u_6^{\,3}\,u_4-103296\,t^3\,u_2\,u_3^{\,5}\,u_5^{\,2}\,u_4^{\,3}\,u_6$$
$$+\,192\,t^5\,u_4^{\,2}\,u_6^{\,3}\,u_3^{\,3}\,u_5^{\,2}-528\,t^5\,u_3^{\,2}\,u_4^{\,3}\,u_6^{\,2}\,u_5^{\,3}-3720\,t^4\,u_2\,u_3^{\,2}\,u_4^{\,3}\,u_5^{\,5}$$
$$-\,136500\,t^2\,u_2^{\,4}\,u_3^{\,3}\,u_5^{\,4}\,u_4^{\,2}-2520\,t^3\,u_2^{\,4}\,u_3^{\,3}\,u_6^{\,3}\,u_5^{\,2}-60\,t^5\,u_4^{\,4}\,u_3^{\,2}\,u_6^{\,3}\,u_5$$
$$-\,7980\,t^3\,u_3^{\,2}\,u_4^{\,2}\,u_6^{\,3}\,u_5\,u_2^{\,4}-48\,t^5\,u_2\,u_3\,u_4\,u_5^{\,6}\,u_6+24576\,t^3\,u_3^{\,7}\,u_5^{\,2}\,u_4^{\,2}\,u_6$$
$$+\,54\,t^5\,u_2^{\,2}\,u_5^{\,2}\,u_4\,u_6^{\,4}\,u_3+24576\,t^2\,u_2\,u_6\,u_3^{\,9}\,u_5^{\,2}-1722\,t^3\,u_2^{\,5}\,u_6^{\,3}\,u_4^{\,3}\,u_5$$
$$+\,17760\,t^4\,u_2\,u_3^{\,2}\,u_4^{\,4}\,u_5^{\,3}\,u_6+336\,t^4\,u_3^{\,2}\,u_4^{\,7}\,u_6\,u_5-159\,t^5\,u_3\,u_4^{\,4}\,u_6\,u_5^{\,4}$$
$$-\,11520\,t^3\,u_2\,u_3^{\,6}\,u_5\,u_4^{\,2}\,u_6^{\,2}-1440\,t^4\,u_2\,u_3^{\,4}\,u_4^{\,2}\,u_6^{\,3}\,u_5+1536\,t^4\,u_4\,u_3^{\,5}\,u_5^{\,4}\,u_6$$
$$+\,1584\,t^4\,u_2\,u_4^{\,8}\,u_6\,u_5-60\,t^5\,u_5\,u_3^{\,2}\,u_2\,u_4^{\,2}\,u_6^{\,4}+168\,t^5\,u_4\,u_6^{\,2}\,u_5^{\,4}\,u_3^{\,3}$$
$$-\,168\,t^5\,u_2\,u_3^{\,2}\,u_4\,u_6^{\,3}\,u_5^{\,3}+1152\,t^4\,u_2\,u_3^{\,5}\,u_6^{\,3}\,u_5^{\,2}+54000\,u_4\,u_2^{\,9}\,u_6^{\,2}\,u_3^{\,2}\,u_5$$
$$+\,1920\,t^4\,u_2\,u_3^{\,4}\,u_4\,u_6^{\,2}\,u_5^{\,3}-73305\,t^2\,u_2^{\,5}\,u_3\,u_5^{\,4}\,u_4^{\,3}+108\,t^5\,u_2\,u_3^{\,2}\,u_6^{\,2}\,u_5^{\,5}$$
$$-\,12240\,t^4\,u_2\,u_3^{\,3}\,u_4^{\,2}\,u_6\,u_5^{\,4}+201\,t^5\,u_2\,u_4^{\,4}\,u_6^{\,2}\,u_5^{\,3}-1920\,t^4\,u_4^{\,4}\,u_6^{\,2}\,u_3^{\,4}\,u_5$$
$$+\,276\,t^3\,u_2^{\,5}\,u_5\,u_3^{\,2}\,u_6^{\,4}-15120\,t^3\,u_2\,u_3^{\,2}\,u_4^{\,8}\,u_5-6\,t^5\,u_2\,u_3\,u_4^{\,2}\,u_6^{\,2}\,u_5^{\,4}$$
$$+\,3456\,t^4\,u_2\,u_3^{\,4}\,u_5^{\,5}\,u_6-141\,t^5\,u_2\,u_4^{\,3}\,u_6\,u_5^{\,5}+206400\,t^2\,u_2\,u_3^{\,6}\,u_5\,u_4^{\,5}$$
$$-\,363\,t^4\,u_2^{\,3}\,u_4^{\,2}\,u_6\,u_5^{\,5}+6600\,t^3\,u_2^{\,3}\,u_4^{\,2}\,u_3^{\,2}\,u_5^{\,5}+48\,t^5\,u_5^{\,2}\,u_2\,u_6^{\,4}\,u_3^{\,3}$$
$$+\,15720\,t^3\,u_2\,u_4^{\,6}\,u_5^{\,2}\,u_3^{\,3}-2304\,t^3\,u_2^{\,2}\,u_3^{\,6}\,u_5\,u_6^{\,3}-12960\,t^3\,u_2\,u_4^{\,7}\,u_6\,u_3^{\,3}$$
$$+\,58884\,t^3\,u_3\,u_2^{\,3}\,u_4^{\,5}\,u_5^{\,2}\,u_6+6384\,t^3\,u_2\,u_4^{\,4}\,u_6^{\,2}\,u_3^{\,5}+22800\,t^3\,u_2^{\,2}\,u_3^{\,3}\,u_5^{\,4}\,u_4^{\,3}$$
$$+\,18912\,t^2\,u_4^{\,2}\,u_2^{\,5}\,u_6^{\,3}\,u_3^{\,3}+18756\,t^2\,u_2^{\,5}\,u_3^{\,2}\,u_4^{\,3}\,u_6^{\,2}\,u_5+91600\,t^2\,u_2^{\,3}\,u_4^{\,6}\,u_3^{\,3}\,u_6$$
$$+\,139320\,t^2\,u_2^{\,4}\,u_4^{\,3}\,u_6\,u_3^{\,3}\,u_5^{\,2}-16186\,t^2\,u_4^{\,3}\,u_2^{\,6}\,u_6^{\,3}\,u_3+7488\,t\,u_2^{\,9}\,u_6^{\,3}\,u_4\,u_5$$
$$+\,314880\,t^2\,u_2^{\,3}\,u_3^{\,5}\,u_5^{\,4}\,u_4+1350\,t^3\,u_3\,u_2^{\,3}\,u_4^{\,4}\,u_5^{\,4}-1020\,t^3\,u_2^{\,4}\,u_3^{\,2}\,u_5^{\,5}\,u_6$$
$$+\,1575\,t^4\,u_2\,u_3\,u_4^{\,5}\,u_5^{\,4}+655\,t^4\,u_4^{\,3}\,u_2^{\,3}\,u_6^{\,2}\,u_5^{\,3}+660\,t^4\,u_2^{\,3}\,u_4^{\,4}\,u_6^{\,3}\,u_5$$
$$+\,5508\,t^2\,u_4^{\,4}\,u_6^{\,2}\,u_2^{\,6}\,u_5-8160\,t^2\,u_2^{\,3}\,u_4^{\,5}\,u_3^{\,3}\,u_5^{\,2}-13428\,t^3\,u_2^{\,3}\,u_4^{\,7}\,u_6\,u_5$$
$$-\,588\,t^3\,u_5\,u_2^{\,6}\,u_6^{\,4}\,u_4-69\,t^4\,u_4\,u_6^{\,5}\,u_3\,u_2^{\,4}+12640\,t^3\,u_2^{\,3}\,u_3^{\,4}\,u_5^{\,3}\,u_6^{\,2}$$
$$-\,93\,t^4\,u_2^{\,2}\,u_4^{\,5}\,u_5^{\,3}\,u_6+195\,t^4\,u_4\,u_6^{\,3}\,u_5^{\,3}\,u_2^{\,4}-14280\,t\,u_2^{\,8}\,u_6^{\,3}\,u_3\,u_4^{\,2}$$
$$-\,166320\,t^2\,u_2^{\,4}\,u_3^{\,4}\,u_5^{\,3}\,u_4\,u_6-15\,t^5\,u_2^{\,2}\,u_5\,u_4^{\,3}\,u_6^{\,4}-9\,t^5\,u_2^{\,2}\,u_4^{\,2}\,u_6^{\,3}\,u_5^{\,3}$$
$$+\,316800\,t\,u_2^{\,5}\,u_4^{\,4}\,u_3^{\,3}\,u_5^{\,2}-319200\,t^2\,u_2^{\,2}\,u_3^{\,4}\,u_4^{\,6}\,u_5-33984\,t^3\,u_2^{\,2}\,u_3^{\,5}\,u_4\,u_6^{\,2}\,u_5^{\,2}$$

$$
\begin{aligned}
&- 91600\, t\, {u_4}^3\, {u_6}^2\, {u_2}^6\, {u_3}^3 + 45600\, t^3\, {u_2}^2\, {u_3}^4\, u_5\, {u_4}^3\, {u_6}^2 + 1665\, t^4\, {u_2}^2\, u_3\, {u_4}^2\, {u_5}^6 \\
&+ 18\, t^5\, u_4\, {u_2}^2\, {u_6}^2\, {u_5}^5 + 230850\, t\, {u_2}^6\, {u_4}^5\, u_3\, {u_5}^2 + 482400\, t^2\, {u_2}^2\, {u_3}^5\, {u_4}^4\, {u_5}^2 \\
&+ 184320\, t\, {u_2}^4\, {u_3}^6\, {u_5}^3\, u_4 - 18912\, t^3\, {u_2}^2\, {u_3}^3\, {u_4}^5\, {u_6}^2 - 6960\, t\, {u_4}^2\, {u_3}^4\, {u_2}^5\, {u_5}^3 \\
&- 9600\, t\, {u_3}^6\, {u_4}^2\, {u_2}^4\, u_6\, u_5 + 12960\, t\, {u_2}^7\, {u_6}^3\, {u_3}^3\, u_4 - 243000\, t\, {u_2}^5\, {u_4}^6\, {u_3}^2\, u_5 \\
&- 201\, t^4\, {u_5}^2\, {u_6}^4\, u_3\, {u_2}^4 - 155520\, t\, {u_2}^5\, u_6\, {u_5}^2\, {u_3}^5\, u_4 + 222000\, t\, {u_4}^5\, {u_3}^4\, {u_2}^4\, u_5 \\
&+ 190920\, t\, {u_2}^6\, {u_4}^2\, {u_5}^2\, {u_3}^3\, u_6 + 31500\, t\, {u_2}^7\, {u_6}^2\, {u_3}^2\, u_5\, {u_4}^2 + 52800\, t\, {u_4}^2\, {u_3}^5\, {u_2}^5\, {u_6}^2 \\
&+ 213120\, t\, u_4\, {u_2}^6\, {u_5}^4\, {u_3}^3 - 9600\, t\, {u_2}^4\, {u_3}^7\, {u_6}^2\, u_4 - 39900\, t^3\, {u_3}^2\, {u_2}^3\, {u_4}^4\, {u_6}^2\, u_5.
\end{aligned}
$$

Generating set $f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}$ for the invariant ring $R_8^{SL_2}$.

$f_2 = -56\,u_3\,u_5 + 28\,u_2\,u_6 + t\,u_8 + 35\,{u_4}^2.$

$f_3 = t\,u_4\,u_8 - 8\,u_2\,u_3\,u_7 - 36\,u_3\,u_4\,u_5 - 22\,u_2\,u_4\,u_6 - 4\,t\,u_5\,u_7 + 15\,{u_4}^3 + 3\,t\,{u_6}^2 + 3\,{u_2}^2\,u_8$
$+ 24\,u_2\,{u_5}^2 + 24\,{u_3}^2\,u_6.$

$f_4 = 168\,{u_2}^2\,u_4\,u_8 + 2408\,{u_3}^2\,{u_5}^2 + 504\,{u_2}^2\,u_5\,u_7 - 1176\,{u_3}^2\,u_4\,u_6 + 168\,t\,u_4\,{u_6}^2$
$- 112\,u_2\,{u_3}^2\,u_8 + 56\,t\,u_2\,{u_7}^2 - 1288\,u_2\,u_3\,u_4\,u_7 + 56\,t\,u_3\,u_5\,u_8 + 56\,t\,u_4\,u_5\,u_7$
$- 168\,t\,u_3\,u_6\,u_7 + 665\,{u_4}^4 + 672\,{u_3}^3\,u_7 - 1624\,u_2\,u_3\,u_5\,u_6 - 42\,t\,{u_4}^2\,u_8$
$+ 112\,{u_2}^2\,{u_6}^2 - 2128\,u_3\,{u_4}^2\,u_5 + t^2\,{u_8}^2 + 3024\,u_2\,{u_4}^2\,u_6 - 112\,t\,{u_5}^2\,u_6 - 1176\,u_2\,u_4\,{u_5}^2.$

$f_5 = -3\,u_6\,{u_3}^2\,{u_4}^2 + {u_6}^3\,t\,u_2 + 2\,{u_5}^3\,u_2\,u_3 - 2\,{u_6}^2\,{u_2}^2\,u_4 + 2\,u_5\,u_6\,{u_3}^3 - u_8\,{u_2}^2\,{u_4}^2 + u_6\,u_8\,{u_2}^3$
$+ 3\,u_6\,u_2\,{u_4}^3 + 4\,u_5\,u_3\,{u_4}^3 - {u_6}^2\,t\,{u_4}^2 - 2\,u_5\,u_8\,t\,u_3\,u_4 - {u_6}^2\,u_2\,{u_3}^2 - 3\,{u_5}^2\,{u_3}^2\,u_4$
$- 3\,{u_5}^2\,u_2\,{u_4}^2 + u_8\,t\,{u_4}^3 - {u_7}^2\,t\,{u_3}^2 + 2\,u_5\,u_6\,u_3\,u_2\,u_4 - {u_5}^4\,t + 2\,u_7\,u_4\,{u_3}^3$
$- {u_5}^2\,u_6\,{u_2}^2 - u_8\,{u_3}^4 - {u_7}^2\,{u_2}^3 + {u_5}^2\,u_8\,t\,u_2 + u_6\,u_8\,t\,{u_3}^2 + {u_7}^2\,t\,u_2\,u_4 - 4\,u_7\,u_3\,u_2\,{u_4}^2$
$+ 3\,u_8\,{u_3}^2\,u_2\,u_4 + 3\,{u_5}^2\,u_6\,t\,u_4 + 2\,{u_5}^2\,u_7\,t\,u_3 - 2\,u_5\,{u_6}^2\,t\,u_3 + 4\,u_5\,u_7\,{u_2}^2\,u_4$
$- 2\,u_5\,u_7\,u_2\,{u_3}^2 - 2\,u_5\,u_7\,t\,{u_4}^2 - 2\,u_5\,u_8\,{u_2}^2\,u_3 + 2\,u_6\,u_7\,{u_2}^2\,u_3 - {u_4}^5 - u_6\,u_8\,t\,u_2\,u_4$
$- 2\,u_5\,u_6\,u_7\,t\,u_2 + 2\,u_6\,u_7\,t\,u_3\,u_4.$

$f_6 = -125\,{u_7}^2\,{u_2}^3\,u_4 - 620\,u_6\,{u_4}^4\,u_2 + 1140\,{u_5}^2\,{u_3}^2\,{u_4}^2 + 69\,{u_5}^2\,u_8\,{u_2}^3 + 660\,{u_5}^2\,u_2\,{u_4}^3$
$- 960\,u_5\,{u_4}^4\,u_3 + 660\,u_6\,{u_3}^2\,{u_4}^3 + 70\,{u_6}^2\,t\,{u_4}^3 + 70\,u_8\,{u_2}^2\,{u_4}^3 + 69\,{u_6}^3\,t\,{u_3}^2$
$+ 126\,{u_7}^2\,{u_2}^2\,{u_3}^2 + 2\,{u_7}^3\,t^2\,u_3 + 14\,{u_5}^2\,{u_7}^2\,t^2 - 20\,u_6\,u_8\,t\,u_2\,{u_4}^2 - 144\,u_5\,u_7\,{u_3}^4$
$+ 90\,u_7\,{u_4}^2\,{u_3}^3 + 76\,{u_5}^3\,{u_3}^3 + 560\,{u_6}^2\,{u_2}^2\,{u_4}^2 + 1815\,u_5\,u_6\,u_2\,u_3\,{u_4}^2 + 654\,{u_6}^2\,{u_3}^4$
$- 150\,u_5\,u_8\,{u_2}^2\,u_3\,u_4 + 6\,{u_6}^4\,t^2 + 135\,u_5\,u_8\,t\,u_3\,{u_4}^2 + 6\,{u_8}^2\,{u_2}^4 - 40\,u_8\,t\,{u_4}^4$
$+ 90\,{u_5}^2\,u_7\,t\,u_3\,u_4 - 5\,u_5\,u_7\,u_8\,t^2\,u_4 - 25\,u_6\,u_8\,t\,{u_3}^2\,u_4 - 45\,u_7\,u_8\,u_2\,u_3\,u_4\,t$
$- 35\,u_6\,u_7\,t\,u_3\,{u_4}^2 + 250\,u_6\,u_7\,{u_2}^2\,u_3\,u_4 + 654\,{u_5}^4\,{u_2}^2 + 10\,u_5\,u_6\,u_7\,t\,{u_3}^2$
$- 160\,{u_6}^3\,{u_2}^3 - 3\,u_6\,u_7\,u_8\,t^2\,u_3 + 40\,u_5\,u_6\,u_7\,t\,u_2\,u_4 - 25\,{u_5}^2\,u_8\,t\,u_2\,u_4 + u_5\,{u_8}^2\,t^2\,u_3$
$- 712\,{u_5}^2\,u_7\,{u_2}^2\,u_3 - 102\,{u_5}^2\,u_8\,t\,{u_3}^2 - 1325\,{u_5}^2\,u_6\,{u_2}^2\,u_4 - 15\,{u_5}^2\,u_6\,t\,{u_4}^2$
$+ 1026\,{u_5}^2\,u_6\,u_2\,{u_3}^2 + 10\,{u_6}^2\,u_8\,t^2\,u_4 - 12\,{u_6}^2\,u_8\,t\,{u_2}^2 - 5\,u_6\,{u_7}^2\,t^2\,u_4$
$+ 21\,u_6\,{u_7}^2\,t\,{u_2}^2 - 4\,{u_8}^2\,t\,{u_3}^2\,u_2 + 10\,{u_8}^2\,t\,{u_2}^2\,u_4 + 24\,u_7\,u_8\,t\,{u_3}^3 - 32\,u_7\,u_8\,{u_2}^3\,u_3$
$- 20\,{u_7}^2\,t\,{u_3}^2\,u_4 + 45\,{u_7}^2\,t\,u_2\,{u_4}^2 - 40\,u_6\,u_8\,{u_2}^3\,u_4 + 64\,u_6\,u_8\,{u_2}^2\,{u_3}^2$
$- 364\,u_6\,u_7\,{u_3}^3\,u_2 - 1325\,{u_6}^2\,u_2\,{u_3}^2\,u_4 - 4\,{u_5}^2\,u_6\,u_8\,t^2 + 264\,u_5\,u_6\,u_7\,{u_2}^3$
$- 40\,{u_6}^3\,t\,u_2\,u_4 - 1710\,{u_5}^3\,u_2\,u_3\,u_4 + 344\,u_5\,{u_6}^2\,{u_2}^2\,u_3 - 16\,u_5\,{u_6}^2\,u_7\,t^2$
$- 68\,{u_5}^3\,u_7\,t\,u_2 + 64\,{u_5}^2\,{u_6}^2\,t\,u_2 + 24\,{u_5}^3\,u_6\,t\,u_3 - 445\,u_7\,{u_4}^3\,u_3\,u_2$
$- 15\,u_8\,{u_4}^2\,{u_3}^2\,u_2 - 35\,u_5\,u_7\,t\,{u_4}^3 - 1710\,u_5\,u_6\,{u_3}^3\,u_4 + 155\,u_5\,u_7\,{u_2}^2\,{u_4}^2$
$+ 24\,u_5\,u_8\,{u_3}^3\,u_2 + 200\,{u_4}^6 + 930\,u_5\,u_7\,u_2\,{u_3}^2\,u_4 - 19\,u_5\,u_7\,u_8\,t\,{u_2}^2$
$+ 117\,u_5\,u_6\,u_8\,t\,u_2\,u_3 - 74\,{u_6}^2\,u_7\,t\,u_2\,u_3 + 10\,u_5\,{u_7}^2\,t\,u_2\,u_3 - 150\,u_5\,{u_6}^2\,t\,u_3\,u_4.$

$$
\begin{aligned}
f_7 = {} & 29400\, u_6^2 u_3^4 u_4 - 75\, u_8^2 u_2^4 u_4 + 31800\, u_5^4 u_2 u_3^2 - 75\, u_6^4 t^2 u_4 + 50\, u_8^2 u_2^3 u_3^2 \\
& + 6000\, u_5^5 t u_3 + 5600\, u_7^2 u_3^4 u_2 - 6875\, u_6^2 t u_4^4 + 3528\, u_5^3 u_7 u_2^3 \\
& + 31800\, u_5^2 u_6 u_3^4 + 3212\, u_6^4 t u_2^2 - 63000\, u_5 u_4^5 u_3 + 3000\, u_8 u_4^5 t + 50\, u_5^2 u_6^3 t^2 \\
& + 6250\, u_7^2 u_2^3 u_4^2 - 1400\, u_6 u_7^2 u_2^4 - 6875\, u_8 u_2^2 u_4^4 + 2\, u_8^3 t^2 u_2^2 \\
& + 31125\, u_5^2 u_2 u_4^4 - 12632\, u_6^3 u_2^3 u_4 + 10792\, u_5^2 u_6^2 u_2^3 - 20375\, u_6 u_4^5 u_2 \\
& - 236\, u_5 u_7 u_8 t u_2 u_3^2 + 150\, u_8^2 t^2 u_4^3 - 16800\, u_6 u_7 u_3^5 - 3750\, u_5^4 t u_4^2 \\
& + 2\, u_6^2 u_8^2 t^3 + 29400\, u_5^4 u_2^2 u_4 + 10792\, u_6^3 u_2^2 u_3^2 + 107325\, u_5^2 u_3^2 u_4^3 \\
& + 3212\, u_6^2 u_8 u_2^4 + 31125\, u_6 u_3^2 u_4^4 - 4072\, u_6 u_7^2 t u_2 u_3^2 - 3750\, u_8 u_4^2 u_3^4 \\
& - 47700\, u_5^3 u_3^3 u_4 - 3000\, u_7 u_4^3 u_3^3 - 24\, u_7 u_8^2 t^2 u_2 u_3 + 22450\, u_6^2 u_2^2 u_4^3 \\
& + 92750\, u_5^2 u_6 u_2 u_3^2 u_4 - 2916\, u_6^2 u_8 t u_2^2 u_4 + 7799\, u_6^2 u_7 t u_2 u_3 u_4 \\
& + 6000\, u_5 u_8 u_3^5 + 6400\, u_5 u_6^2 t u_3 u_4^2 - 5020\, u_5^3 u_8 t u_2 u_3 + 9648\, u_5 u_6 u_7 u_2^2 u_3^2 \\
& - 4450\, u_5 u_6 u_7 u_2^3 u_4 + 641\, u_6 u_7 u_8 t u_2^2 u_3 - 4496\, u_5 u_7^2 t u_2 u_3 u_4 \\
& - 377\, u_5 u_6^2 u_8 t^2 u_3 - 5020\, u_5 u_6 u_8 u_3^3 t - 6071\, u_5 u_6^2 u_7 t u_2^2 \\
& + 9325\, u_6 u_7 t u_4^3 u_3 + 1825\, u_7 u_8 u_2^3 u_3 u_4 + 2068\, u_5^2 u_7 u_2^2 u_3 u_4 \\
& + 680\, u_5^2 u_7 u_8 t^2 u_3 - 22200\, u_5^3 u_6 t u_3 u_4 + 2175\, u_6 u_8 u_2^2 u_3^2 u_4 \\
& - 10725\, u_5 u_8 t u_4^3 u_3 - 425\, u_5 u_7 u_8 t^2 u_4^2 - 13\, u_6 u_7 u_8 t^2 u_3 u_4 \\
& + 2208\, u_6^2 u_8 t u_2 u_3^2 + 50202\, u_5 u_6^2 u_2^2 u_3 u_4 + 15200\, u_5^2 u_7 t u_3 u_4^2 + 2\, u_7^4 t^3 \\
& + 2175\, u_5^2 u_6^2 t u_2 u_4 + 11250\, u_4^7 - 15000\, u_7^2 u_2^2 u_3^2 u_4 - 200\, u_6 u_8 u_3^4 u_2 \\
& + 3555\, u_6 u_8 u_2^3 u_4^2 - 50300\, u_6^2 u_2 u_3^2 u_4^2 - 6375\, u_5 u_7 t u_4^4 + 12725\, u_5 u_7 u_2^2 u_4^3 \\
& - 95400\, u_5 u_6 u_3^3 u_4^2 + 13200\, u_5 u_7 u_3^4 u_4 + 13000\, u_8 u_3^2 u_4^3 u_2 \\
& - 2875\, u_7 u_4^4 u_3 u_2 + 5080\, u_5 u_7^2 u_2^3 u_3 + 5728\, u_5 u_7^2 u_3^3 t + 161\, u_6 u_8^2 u_2^3 t \\
& - 2032\, u_6^2 u_7 u_2^3 u_3 + 1812\, u_6^2 u_7 u_3^3 t - 245\, u_7^2 u_8 u_2^3 t + 245\, u_6^2 u_8 t^2 u_4^2 \\
& - 19400\, u_5^2 u_7 u_3^3 u_2 - 51400\, u_5 u_6^2 u_3^3 u_2 - 4053\, u_6^3 t u_3^2 u_4 + 3555\, u_6^3 t u_2 u_4^2 \\
& + 180\, u_6 u_7^2 t^2 u_4^2 - 72\, u_7^3 t^2 u_3 u_4 + 520\, u_7^3 t u_2^2 u_3 - 50300\, u_5^2 u_6 u_2^2 u_4^2 \\
& + 13555\, u_5^2 u_8 u_2^2 u_3^2 - 95400\, u_5^3 u_2 u_3 u_4^2 + 13000\, u_5^2 u_6 t u_4^3 \\
& - 4053\, u_5^2 u_8 u_2^3 u_4 + 245\, u_8^2 u_2^2 u_4^2 t - 855\, u_5 u_7 u_8 u_2^4 - 900\, u_7 u_8 u_3^3 u_2^2 \\
& + 2275\, u_7^2 t u_4^3 u_2 - 3230\, u_7^2 u_3^2 u_4^2 t + 124\, u_7^3 u_5 t^2 u_2 - 51400\, u_5^3 u_6 u_2^2 u_3 \\
& + 13555\, u_5^2 u_6^2 t u_3^2 - 51\, u_6^2 u_7^2 t^2 u_2 - 7780\, u_5^3 u_7 t u_3^2 + 70\, u_5^2 u_8^2 t^2 u_2 \\
& + 165\, u_6^3 u_7 t^2 u_3 + 161\, u_6^3 u_8 t^2 u_2 + 1596\, u_5^2 u_7^2 t u_2^2 + 2\, u_7^2 u_8 t^2 u_3^2 \\
& + 70\, u_6 u_8^2 t^2 u_3^2 - 200\, u_5^4 u_6 t u_2 - 4\, u_6 u_7^2 u_8 t^3 - 1400\, u_6 u_8 t u_4^3 u_2 \\
& + 3125\, u_6 u_8 u_3^2 u_4^2 t + 43800\, u_6 u_7 u_3^3 u_4 u_2 - 31230\, u_6 u_7 u_2^2 u_3 u_4^2 \\
& + 6400\, u_5 u_8 u_2^2 u_3 u_4^2 - 22200\, u_5 u_8 u_3^3 u_4 u_2 - 6200\, u_5 u_7 u_2 u_3^2 u_4^2 \\
& + 48175\, u_5 u_6 u_2 u_3 u_4^3 - 377\, u_5 u_8^2 t u_2^2 u_3 + 1150\, u_6 u_7^2 t u_2^2 u_4 \\
& + 9475\, u_5^2 u_8 t u_3^2 u_4 + 3125\, u_5^2 u_8 t u_2 u_4^2 - 16552\, u_5 u_6 u_7 t u_3^2 u_4 \\
& - 10530\, u_5 u_6 u_7 t u_2 u_4^2 + 1667\, u_5 u_6 u_8 t u_2 u_3 u_4 - 10853\, u_5 u_6 u_8 u_2^3 u_3 \\
& + 16\, u_5 u_7 u_8 t u_2^2 u_4 + 35\, u_7 u_8 u_2 u_3 u_4^2 t + 13668\, u_5^2 u_6 u_7 t u_2 u_3 \\
& - 228\, u_5 u_6 u_7^2 t^2 u_3 + 2208\, u_5^2 u_6 u_8 t u_2^2 + 100\, u_5^3 u_7 t u_2 u_4 \\
& - 10853\, u_6^3 u_5 t u_2 u_3 - 75\, u_5 u_6^2 u_7 t^2 u_4 + 23\, u_6 u_8^2 t^2 u_2 u_4 + 37\, u_7^2 u_8 t^2 u_2 u_4 \\
& - 275\, u_5 u_8^2 t^2 u_3 u_4 - 344\, u_5 u_7 u_8 u_6 t^2 u_2.
\end{aligned}
$$

$$
\begin{aligned}
f_8 = {} & 480\, u_6\, {u_4}^5\, {u_3}^2 + 114\, {u_5}^4\, {u_6}^2\, t^2 - 25\, {u_5}^4\, t\, {u_4}^3 - 1056\, {u_5}^2\, {u_3}^2\, {u_4}^4 - {u_7}^4\, t^3\, u_4 - {u_6}^3\, {u_7}^2\, t^3 \\
& + 3741\, {u_6}^2\, {u_2}^2\, {u_4}^4 + 114\, {u_8}^2\, {u_3}^4\, {u_2}^2 + 480\, {u_5}^2\, {u_4}^5\, u_2 + 74\, {u_6}^2\, u_7\, u_8\, t^2\, u_2\, u_3 \\
& + 1368\, {u_5}^3\, {u_3}^3\, {u_4}^2 + 1536\, u_5\, {u_6}^2\, {u_3}^5 - 888\, {u_5}^2\, u_7\, {u_3}^5 - 40\, {u_5}^6\, t\, u_2 - 40\, u_6\, u_8\, {u_3}^6 \\
& + 174\, {u_8}^2\, {u_2}^4\, {u_4}^2 - 940\, u_7\, {u_4}^4\, {u_3}^3 - 365\, u_8\, {u_4}^5\, {u_2}^2 + 1371\, {u_6}^2\, {u_3}^4\, {u_4}^2 + 180\, u_8\, {u_4}^6\, t \\
& + {u_6}^4\, u_8\, t^3 + 45\, {u_7}^2\, u_8\, {u_2}^5 + 1521\, {u_5}^2\, {u_7}^2\, {u_2}^4 + 4\, u_5\, {u_8}^2\, u_7\, t^2\, {u_2}^2 \\
& + 1076\, {u_6}^3\, u_5\, t\, u_2\, u_3\, u_4 - 66\, {u_6}^5\, t^2\, u_2 + 1536\, {u_5}^5\, {u_2}^2\, u_3 + 320\, u_5\, {u_4}^6\, u_3 \\
& + 42\, {u_5}^3\, u_7\, u_8\, t^2\, u_2 + 174\, {u_6}^4\, t^2\, {u_4}^2 - 160\, {u_5}^5\, u_7\, t^2 + 100\, {u_7}^2\, {u_2}^3\, {u_4}^3 \\
& - 190\, u_5\, u_6\, u_8\, {u_3}^3\, {u_2}^2 + {u_8}^3\, {u_2}^4\, t - 3728\, {u_6}^3\, {u_2}^3\, {u_4}^2 - 120\, {u_7}^3\, {u_2}^4\, u_3 \\
& - 1228\, u_5\, {u_6}^2\, u_7\, u_2\, {u_3}^2\, t + 1371\, {u_5}^4\, {u_2}^2\, {u_4}^2 - 25\, u_8\, {u_4}^3\, {u_3}^4 + 14\, {u_8}^2\, t^2\, {u_4}^4 \\
& - 34\, u_6\, {u_8}^2\, {u_2}^3\, t\, u_4 + 6\, {u_7}^4\, t^2\, {u_2}^2 - 768\, {u_5}^4\, u_6\, {u_2}^3 - 66\, u_6\, {u_8}^2\, {u_2}^5 \\
& - 6\, u_6\, u_7\, u_8\, t^2\, u_3\, {u_4}^2 + 2\, u_6\, {u_7}^2\, u_8\, t^3\, u_4 - 960\, u_6\, {u_4}^6\, u_2 - 12\, u_6\, {u_7}^2\, u_8\, t^2\, {u_2}^2 \\
& - 708\, u_5\, u_8\, {u_3}^3\, u_2\, {u_4}^2 + 784\, {u_7}^2\, {u_3}^6 + 9336\, {u_5}^2\, u_6\, u_2\, {u_3}^2\, {u_4}^2 - 37\, {u_8}^2\, t\, u_2\, {u_3}^2\, {u_4}^2 \\
& - 768\, {u_6}^3\, {u_3}^4\, u_2 - 365\, {u_6}^2\, {u_4}^5\, t + 936\, {u_5}^2\, u_6\, u_8\, u_2\, {u_3}^2\, t + 486\, {u_5}^3\, u_6\, u_7\, t^2\, u_4 \\
& + 18\, u_5\, {u_7}^2\, u_8\, t^2\, u_2\, u_3 - 858\, u_5\, u_8\, t\, {u_4}^4\, u_3 + 474\, u_7\, u_8\, {u_2}^2\, {u_3}^3\, u_4 - 50\, {u_7}^3\, u_6\, t^2\, u_3\, u_2 \\
& + 1172\, u_5\, u_8\, {u_2}^2\, {u_4}^3\, u_3 + 614\, {u_5}^4\, u_6\, t\, u_2\, u_4 + 614\, u_6\, u_8\, {u_3}^4\, u_2\, u_4 \\
& - 134\, {u_5}^3\, u_8\, u_6\, t^2\, u_3 - 352\, {u_5}^4\, u_7\, t\, u_2\, u_3 + 369\, {u_5}^4\, {u_3}^4 - 10\, u_7\, {u_8}^2\, {u_2}^3\, t\, u_3 \\
& + 100\, {u_7}^3\, u_5\, t^2\, u_2\, u_4 + 1232\, u_7\, u_8\, t\, u_2\, u_3\, {u_4}^3 + 1076\, u_5\, u_6\, u_8\, {u_2}^3\, u_3\, u_4 \\
& - 62\, u_5\, {u_8}^2\, t^2\, u_3\, {u_4}^2 + 474\, {u_5}^3\, u_7\, {u_3}^2\, u_4\, t + 182\, u_7\, u_8\, {u_2}^3\, u_3\, {u_4}^2 \\
& - 198\, {u_6}^2\, u_8\, {u_2}^2\, {u_4}^2\, t - 78\, u_5\, u_6\, u_7\, u_8\, t^2\, {u_3}^2 - 46\, u_5\, {u_8}^2\, u_6\, t^2\, u_2\, u_3 \\
& - 5174\, u_5\, {u_6}^2\, {u_3}^3\, u_2\, u_4 - 50\, u_5\, u_7\, u_8\, {u_2}^3\, {u_3}^2 + 88\, u_5\, {u_7}^2\, u_6\, t^2\, u_3\, u_4 \\
& + 824\, {u_5}^3\, u_8\, t\, u_2\, u_3\, u_4 - 578\, u_7\, u_8\, {u_3}^3\, {u_4}^2\, t - 7864\, u_5\, u_6\, u_7\, {u_2}^2\, {u_3}^2\, u_4 \\
& + 66\, {u_7}^2\, u_8\, {u_2}^3\, t\, u_4 - 1217\, {u_5}^2\, {u_6}^2\, u_2\, {u_4}^2\, t + 1290\, u_5\, {u_6}^2\, {u_2}^2\, {u_4}^2\, u_3 \\
& + 142\, u_6\, u_7\, u_8\, {u_2}^4\, u_3 - 186\, {u_5}^2\, {u_7}^2\, u_6\, t^2\, u_2 + 286\, {u_6}^3\, u_7\, {u_2}^2\, u_3\, t \\
& - 666\, u_5\, u_7\, u_8\, {u_2}^4\, u_4 + 644\, u_5\, u_7\, u_8\, {u_3}^4\, t + 448\, u_5\, {u_7}^2\, u_6\, {u_2}^2\, u_3\, t \\
& - 1058\, u_5\, {u_6}^2\, u_8\, {u_2}^2\, u_3\, t + 86\, u_5\, u_7\, u_8\, t^2\, {u_4}^3 + 310\, u_6\, u_7\, t\, {u_4}^4\, u_3 \\
& - 118\, u_6\, {u_7}^2\, t\, u_2\, {u_3}^2\, u_4 - 134\, u_5\, {u_8}^2\, {u_3}^3\, u_2\, t + 1344\, {u_5}^2\, u_8\, {u_3}^2\, {u_4}^2\, t \\
& - 190\, {u_5}^3\, {u_6}^2\, t\, u_2\, u_3 + 1168\, {u_6}^4\, {u_2}^4 + {u_8}^3\, {u_3}^2\, t^2\, u_2 - {u_8}^3\, t^2\, {u_2}^2\, u_4 - {u_5}^2\, {u_7}^2\, u_8\, t^3 \\
& + {u_5}^2\, {u_8}^2\, u_6\, t^3 - {u_6}^2\, {u_8}^2\, t^3\, u_4 + 240\, u_8\, {u_4}^4\, {u_3}^2\, u_2 + 2190\, u_7\, {u_4}^5\, u_3\, u_2 \\
& - 3032\, {u_7}^2\, {u_3}^4\, u_2\, u_4 + 54\, u_6\, u_8\, {u_2}^3\, {u_4}^3 - 122\, {u_8}^2\, t\, {u_4}^3\, {u_2}^2 - 2664\, u_6\, u_7\, {u_3}^5\, u_4 \\
& + 2919\, {u_7}^2\, {u_2}^2\, {u_3}^2\, {u_4}^2 - 1095\, {u_7}^2\, t\, {u_4}^4\, u_2 + 595\, {u_7}^2\, t\, {u_4}^3\, {u_3}^2 - 342\, {u_8}^2\, {u_2}^3\, {u_3}^2\, u_4 \\
& - 730\, u_5\, u_7\, {u_2}^2\, {u_4}^4 + 3048\, u_5\, u_7\, {u_3}^4\, {u_4}^2 + 60\, u_5\, u_8\, {u_3}^5\, u_4 - 30\, u_5\, u_7\, {u_4}^5\, t \\
& - 4268\, {u_6}^2\, u_2\, {u_4}^3\, {u_3}^2 - 604\, u_5\, u_6\, {u_3}^3\, {u_4}^3 - 604\, {u_5}^3\, u_2\, {u_4}^3\, u_3 + 16\, {u_7}^3\, {u_3}^3\, t\, u_2 \\
& + 240\, {u_5}^2\, u_6\, t\, {u_4}^4 + 40\, {u_8}^2\, {u_3}^4\, t\, u_4 - 248\, u_7\, u_8\, {u_3}^5\, u_2 - 122\, {u_6}^2\, u_8\, t^2\, {u_4}^3 \\
& + 130\, u_6\, {u_7}^2\, t^2\, {u_4}^3 + 1848\, u_5\, {u_7}^2\, {u_3}^3\, {u_2}^2 + 132\, u_5\, {u_8}^2\, {u_2}^4\, u_3 + 108\, {u_5}^2\, u_8\, {u_3}^4\, u_2 \\
& - 384\, {u_5}^2\, u_8\, {u_2}^3\, {u_4}^2 - 3054\, {u_5}^2\, u_6\, {u_3}^4\, u_4 - 4268\, {u_5}^2\, u_6\, {u_2}^2\, {u_4}^3 - 52\, {u_7}^3\, t^2\, u_3\, {u_4}^2 \\
& + 4138\, {u_6}^3\, {u_2}^2\, {u_3}^2\, u_4 + 54\, {u_6}^3\, t\, {u_4}^3\, u_2 - 384\, {u_6}^3\, {u_3}^2\, {u_4}^2\, t + 370\, u_6\, {u_7}^2\, {u_2}^4\, u_4 \\
& - 48\, u_6\, {u_7}^2\, {u_3}^4\, t - 16\, u_6\, {u_7}^2\, {u_2}^3\, {u_3}^2 - 187\, {u_6}^2\, u_8\, {u_2}^3\, {u_3}^2 - 6\, u_7\, {u_8}^2\, {u_3}^3\, t^2 \\
& - 69\, {u_6}^3\, u_8\, t^2\, {u_3}^2 - 36\, {u_7}^3\, u_5\, t^2\, {u_3}^2 - 50\, {u_4}^8 + 966\, u_5\, u_6\, u_2\, {u_4}^4\, u_3 \\
& - 7452\, u_5\, u_7\, u_2\, {u_4}^3\, {u_3}^2 - 1217\, u_6\, u_8\, {u_2}^2\, {u_3}^2\, {u_4}^2 + 304\, u_6\, u_8\, t\, {u_4}^4\, u_2 \\
& + 44\, u_6\, u_8\, t\, {u_4}^3\, {u_3}^2 - 5814\, u_6\, u_7\, {u_2}^2\, {u_4}^3\, u_3 - 166\, {u_7}^3\, t\, {u_2}^2\, u_3\, u_4 \\
& + 8662\, u_6\, u_7\, {u_3}^3\, u_2\, {u_4}^2 + 972\, {u_6}^2\, u_7\, {u_3}^3\, t\, u_4 + 1496\, {u_6}^2\, u_8\, t\, u_2\, {u_3}^2\, u_4 \\
& + 654\, {u_5}^2\, u_7\, {u_3}^3\, u_2\, u_4 + 1372\, u_5\, {u_7}^2\, t\, u_2\, u_3\, {u_4}^2 - 4034\, u_5\, {u_7}^2\, {u_2}^3\, u_3\, u_4 \\
& - 740\, u_5\, {u_7}^2\, {u_3}^3\, t\, u_4 + 364\, u_5\, {u_8}^2\, t\, {u_2}^2\, u_3\, u_4 + 1172\, u_5\, {u_6}^2\, t\, {u_4}^3\, u_3 \\
& + 24\, {u_5}^2\, u_7\, t\, {u_4}^3\, u_3 + 8190\, {u_5}^2\, u_7\, {u_2}^2\, {u_4}^2\, u_3 - 39\, {u_5}^2\, u_8\, {u_2}^2\, {u_3}^2\, u_4 \\
& + 44\, {u_5}^2\, u_8\, t\, {u_4}^3\, u_2 + 1640\, u_5\, u_6\, u_7\, {u_3}^4\, u_2 + 3122\, u_5\, u_6\, u_7\, {u_2}^3\, {u_4}^2 \\
& + 32\, u_6\, {u_8}^2\, t^2\, u_2\, {u_4}^2 - 37\, {u_7}^2\, u_8\, t^2\, u_2\, {u_4}^2 + 39\, {u_7}^2\, u_8\, t^2\, {u_3}^2\, u_4 + 108\, u_6\, u_7\, u_8\, {u_3}^3\, t\, u_2 \\
& + 824\, u_5\, u_6\, u_8\, {u_3}^3\, t\, u_4 - 1404\, u_5\, u_7\, u_8\, t\, u_2\, {u_3}^2\, u_4 - 3136\, u_5\, u_6\, u_8\, t\, u_2\, u_3\, {u_4}^2 \\
& + 1928\, u_5\, u_6\, u_7\, t\, {u_4}^3\, u_2 - 2136\, u_5\, u_6\, u_7\, {u_3}^2\, {u_4}^2\, t - 314\, u_5\, u_7\, u_8\, {u_2}^2\, {u_4}^2\, t -
\end{aligned}
$$

$$
\begin{aligned}
&- 200\, u_6\, u_7\, u_8\, t\, {u_2}^2\, u_3\, u_4 + 13\, {u_7}^2\, u_8\, {u_2}^2\, {u_3}^2\, t + 2\, u_7\, {u_8}^2\, t^2\, u_2\, u_4\, u_3 \\
&+ 583\, u_6\, {u_7}^2\, {u_2}^2\, {u_4}^2\, t + 790\, {u_5}^2\, u_6\, u_7\, {u_3}^3\, t + 3662\, {u_5}^2\, u_6\, u_7\, {u_2}^3\, u_3 \\
&+ 188\, {u_5}^2\, u_6\, u_7\, t\, u_2\, u_3\, u_4 - 602\, {u_5}^3\, u_7\, u_2\, {u_4}^2\, t - 5174\, {u_5}^3\, u_6\, {u_2}^2\, u_3\, u_4 \\
&- 198\, {u_6}^3\, u_7\, t^2\, u_3\, u_4 - 34\, {u_6}^3\, u_8\, t^2\, u_2\, u_4 + 4138\, {u_5}^2\, {u_6}^2\, {u_2}^3\, u_4 + 1681\, {u_5}^2\, {u_6}^2\, {u_2}^2\, {u_3}^2 \\
&- 3054\, {u_5}^4\, u_2\, {u_3}^2\, u_4 + 32\, {u_6}^4\, {u_2}^2\, u_4\, t + 105\, {u_6}^4\, u_2\, {u_3}^2\, t + 148\, {u_6}^3\, u_8\, {u_2}^3\, t \\
&- 210\, {u_6}^3\, u_5\, {u_3}^3\, t - 2904\, {u_6}^3\, u_5\, {u_2}^3\, u_3 + 1298\, {u_5}^3\, u_6\, {u_3}^3\, u_2 - 2682\, {u_5}^3\, u_7\, {u_2}^3\, u_4 \\
&- 1594\, {u_5}^3\, u_7\, {u_2}^2\, {u_3}^2 - 1078\, {u_5}^3\, u_8\, {u_3}^3\, t - 210\, {u_5}^3\, u_8\, {u_2}^3\, u_3 - 2\, {u_7}^3\, u_5\, {u_2}^3\, t \\
&+ 59\, {u_5}^2\, {u_8}^2\, t^2\, {u_3}^2 - 2760\, u_5\, {u_6}^2\, u_7\, {u_2}^4 + 105\, {u_5}^2\, u_6\, u_8\, {u_2}^4 - 244\, {u_5}^2\, {u_7}^2\, t^2\, {u_4}^2 \\
&+ 3\, {u_6}^2\, {u_8}^2\, t^2\, {u_2}^2 + 102\, {u_6}^2\, {u_7}^2\, t^2\, {u_3}^2 - 1048\, {u_6}^2\, u_7\, {u_3}^3\, {u_2}^2 + 32\, {u_6}^2\, u_8\, {u_2}^4\, u_4 \\
&- 621\, {u_6}^2\, u_8\, {u_3}^4\, t + 108\, {u_5}^4\, u_6\, t\, {u_3}^2 - 342\, {u_6}^3\, {u_5}^2\, t^2\, u_4 - 187\, {u_6}^3\, {u_5}^2\, t\, {u_2}^2 \\
&+ 60\, {u_5}^5\, t\, u_3\, u_4 + 2\, {u_7}^3\, u_5\, u_6\, t^3 + 40\, {u_5}^4\, u_8\, t^2\, u_4 - 621\, {u_5}^4\, u_8\, t\, {u_2}^2 + 132\, {u_6}^4\, u_5\, t^2\, u_3 \\
&+ 196\, {u_5}^3\, {u_7}^2\, t^2\, u_3 - 69\, {u_5}^2\, {u_8}^2\, {u_2}^3\, t - 135\, {u_6}^2\, {u_7}^2\, {u_2}^3\, t + 364\, u_5\, {u_6}^2\, u_8\, t^2\, u_3\, u_4 \\
&- 94\, {u_5}^2\, u_7\, u_8\, t^2\, u_3\, u_4 - 16\, {u_6}^2\, {u_7}^2\, t^2\, u_2\, u_4 + 32\, u_5\, u_6\, u_7\, u_8\, {u_2}^3\, t \\
&- 40\, u_5\, u_6\, u_7\, u_8\, t^2\, u_2\, u_4 - 39\, {u_5}^2\, {u_6}^2\, {u_3}^2\, u_4\, t - 692\, {u_5}^2\, {u_7}^2\, {u_2}^2\, u_4\, t \\
&+ 230\, {u_5}^2\, {u_7}^2\, u_2\, {u_3}^2\, t - 978\, {u_6}^2\, u_7\, t\, u_2\, u_3\, {u_4}^2 + 2788\, {u_6}^2\, u_7\, {u_2}^3\, u_3\, u_4 \\
&- 134\, u_5\, {u_6}^2\, u_7\, t^2\, {u_4}^2 - 702\, u_5\, {u_6}^2\, u_7\, {u_2}^2\, u_4\, t - 37\, {u_5}^2\, u_6\, u_8\, t^2\, {u_4}^2 \\
&+ 332\, {u_5}^2\, u_7\, u_8\, {u_2}^2\, u_3\, t + 1496\, {u_5}^2\, u_6\, u_8\, {u_2}^2\, u_4\, t - 250\, {u_5}^2\, {u_6}^2\, u_7\, t^2\, u_3 \\
&+ 712\, {u_5}^3\, u_6\, u_7\, t\, {u_2}^2 - 708\, {u_5}^3\, u_6\, u_3\, {u_4}^2\, t - 2\, u_5\, {u_6}^2\, u_7\, u_8\, t^3 + 206\, {u_6}^3\, u_5\, u_7\, t^2\, u_2.
\end{aligned}
$$

$$
\begin{aligned}
f_9 = {} & 24\, u_6^2\, u_7^2\, u_2^5 - 159\, u_6^4\, u_3^4\, t + 95\, u_8\, u_4^6\, u_2^2 + 10\, u_7^3\, u_5\, u_2^5 + 136\, u_7^3\, u_3^5\, t \\
& + 66\, u_5^6\, t\, u_3^2 + 15\, u_7^4\, u_2^4\, t - 1670\, u_5^3\, u_7\, u_2^2\, u_3^2\, u_4 - 732\, u_6^3\, u_2^3\, u_4^3 \\
& - 8\, u_7\, u_8^2\, t^2\, u_3^3\, u_4 + 75\, u_5^4\, t\, u_4^4 + 40\, u_8^2\, u_2^4\, u_4^3 + 280\, u_6^4\, u_2^3\, u_3^2 + 78\, u_5^2\, u_8^2\, u_2^5 \\
& + u_7^4\, t^3\, u_4^2 - 420\, u_5^3\, u_6\, u_3^5 + 40\, u_6^4\, t^2\, u_4^3 - 420\, u_7\, u_4^5\, u_3^3 + 3456\, u_5^2\, u_3^2\, u_4^5 \\
& + 78\, u_6^5\, t^2\, u_3^2 - 3000\, u_5^3\, u_3^3\, u_4^3 + 72\, u_6^3\, u_8\, u_2^5 + 75\, u_8\, u_4^4\, u_3^4 + 1200\, u_5^4\, u_2^2\, u_4^3 \\
& + u_5^4\, u_8^2\, t^3 + 240\, u_7^2\, u_3^6\, u_4 + 66\, u_5^2\, u_8\, u_3^6 + 24\, u_7^3\, u_2^3\, u_3^3 - 50\, u_7^2\, u_2^3\, u_4^4 \\
& + 960\, u_5^2\, u_4^6\, u_2 + 1314\, u_6^2\, u_4^5\, u_2^2 + 960\, u_6\, u_4^6\, u_3^2 + 880\, u_6\, u_8\, u_2^2\, u_3^2\, u_4^3 \\
& - 1126\, u_6^2\, u_8\, u_2\, u_3^2\, u_4^2\, t - 159\, u_5^4\, u_8\, u_2^4 + 630\, u_5^4\, u_3^4\, u_4 + 734\, u_5\, u_6\, u_8\, u_2^2\, u_3^3\, u_4 \\
& - 58\, u_6^2\, u_7^2\, t^2\, u_2\, u_4^2 - 120\, u_7\, u_8\, u_3^7 - 36\, u_5^3\, u_6\, u_7\, t\, u_2^2\, u_4 + 276\, u_5\, u_6\, u_7\, u_2\, u_3^4\, u_4 \\
& + 20\, u_8^2\, u_3^6\, u_2 + 16\, u_8^2\, t^2\, u_4^5 - 760\, u_5^4\, u_6\, t\, u_2\, u_4^2 - 268\, u_6^3\, u_8\, t\, u_2^3\, u_4 + 40\, u_8\, u_4^7\, t \\
& - 1440\, u_5\, u_4^7\, u_3 + 1200\, u_6^2\, u_3^4\, u_4^3 - 250\, u_5\, u_6^2\, u_2\, u_3^3\, u_4^2 - 32\, u_5^2\, u_8^2\, t\, u_2^3\, u_4 \\
& + 72\, u_6^5\, u_2^3\, t + 360\, u_5^3\, u_8\, u_2^3\, u_3\, u_4 - 60\, u_5^3\, u_7\, u_8\, t^2\, u_3^2 + u_8^3\, u_3^4\, t^2 - 420\, u_5^5\, u_3^3\, u_2 \\
& + 112\, u_6^4\, u_2^4\, u_4 + 62\, u_7^3\, u_5\, t^2\, u_3^2\, u_4 + 544\, u_6^2\, u_7\, t\, u_2\, u_4^3\, u_3 + 10\, u_7^2\, u_8\, t\, u_2\, u_3^4 \\
& + 20\, u_5^6\, u_6\, t^2 - 880\, u_6\, u_4^7\, u_2 + 168\, u_5^4\, u_6\, t\, u_3^2\, u_4 + 170\, u_7\, u_8\, u_2^3\, u_3\, u_4^3 \\
& + 16\, u_5^2\, u_7^2\, u_8\, t^2\, u_2^2 + 80\, u_5^6\, u_2^3 + 95\, u_6^2\, u_4^6\, t - 222\, u_5\, u_8\, t\, u_3\, u_4^5 + 80\, u_6^3\, u_3^6 \\
& + 50\, u_5^3\, u_7\, t\, u_2\, u_4^3 + 280\, u_6^3\, u_5^2\, u_2^4 + u_8^3\, u_2^6 + 30\, u_5^2\, u_7\, u_2\, u_3^3\, u_4^2 + u_6^6\, t^3 \\
& - 304\, u_5\, u_6^2\, u_8\, t\, u_2\, u_3^3 - 4\, u_7^2\, u_8\, t\, u_2^2\, u_3^2\, u_4 + 230\, u_5^2\, u_6\, u_7\, t\, u_3^3\, u_4 \\
& - 234\, u_6\, u_7\, u_8\, u_3^5\, t + 350\, u_6^4\, t\, u_2\, u_3^2\, u_4 + 110\, u_5^2\, u_6\, u_8\, t^2\, u_4^3 + 110\, u_6^3\, u_7\, t^2\, u_3\, u_4^2 \\
& + 234\, u_7^3\, u_5\, t\, u_2^2\, u_3^2 - 204\, u_6^4\, u_5\, t^2\, u_3\, u_4 - 46\, u_5\, u_6\, u_8^2\, t\, u_2^3\, u_3 \\
& - 494\, u_7^3\, t\, u_2\, u_3^3\, u_4 - 206\, u_5\, u_8^2\, t\, u_2\, u_3^3\, u_4 + 90\, u_5^3\, u_8\, t\, u_3^3\, u_4 - 76\, u_7^3\, u_5^2\, t^2\, u_2\, u_3 \\
& - 50\, u_5\, u_8^2\, t^2\, u_3\, u_4^3 - 26\, u_6^4\, u_7\, t^2\, u_2\, u_3 + 2754\, u_5\, u_6\, u_2\, u_3\, u_4^5 - 390\, u_6^3\, u_5^2\, t\, u_2^2\, u_4 \\
& - 96\, u_5^3\, u_6^2\, u_7\, t^2\, u_2 - 558\, u_6\, u_7^2\, u_2^3\, u_3^2\, u_4 + 24\, u_6^2\, u_7^2\, t^2\, u_3^2\, u_4 \\
& + 90\, u_6^2\, u_7\, u_8\, t\, u_2^3\, u_3 + 252\, u_6^2\, u_7^2\, t\, u_2^3\, u_4 + 2\, u_5^2\, u_7^2\, u_8\, t^3\, u_4 \\
& + 944\, u_6^2\, u_7\, u_2^3\, u_3\, u_4^2 - 2\, u_5^2\, u_6\, u_8^2\, t^3\, u_4 - 8\, u_7\, u_8^2\, t\, u_2^2\, u_3^3 - 468\, u_5^2\, u_6\, u_7\, u_2^2\, u_3^3 \\
& - 204\, u_5\, u_8^2\, u_2^4\, u_3\, u_4 - 230\, u_5\, u_7\, u_8\, u_2^4\, u_4^2 + 180\, u_5^2\, u_7\, u_8\, t^2\, u_3\, u_4^2 \\
& - 106\, u_5^4\, u_7\, u_6\, t^2\, u_3 - 82\, u_5^3\, u_7\, u_8\, u_2^3\, t - 2460\, u_5\, u_7\, u_2\, u_3^2\, u_4^4 \\
& - 390\, u_6^2\, u_8\, u_2^3\, u_3^2\, u_4 + 758\, u_5^2\, u_6\, u_7\, u_2^3\, u_3\, u_4 + 20\, u_7^2\, u_8\, t\, u_2^3\, u_4^2 \\
& - 54\, u_7^3\, u_6\, t\, u_2^3\, u_3 + 6\, u_6^2\, u_8^2\, t^2\, u_2^2\, u_4 + 6\, u_7^2\, u_8\, u_5\, t\, u_2^3\, u_3 - 310\, u_5^5\, u_6\, t\, u_2\, u_3 \\
& - 68\, u_5^2\, u_7\, u_8\, u_2^4\, u_3 + 2220\, u_5^2\, u_6\, u_2\, u_3^2\, u_4^3 - 1750\, u_6\, u_7\, u_2^2\, u_3\, u_4^4 \\
& + 16\, u_7^2\, u_8\, t^2\, u_3^2\, u_4^2 + 2670\, u_6\, u_7\, u_2\, u_3^3\, u_4^3 - 610\, u_6\, u_7^2\, t\, u_2^2\, u_4^3 \\
& + 370\, u_7^3\, t\, u_2^2\, u_4^2\, u_3 - 654\, u_5\, u_6\, u_8\, u_2^3\, u_3\, u_4^2 - 130\, u_7\, u_8\, t\, u_2\, u_4^4\, u_3 \\
& + 298\, u_5\, u_6\, u_7^2\, u_2^4\, u_3 - 12\, u_6\, u_8^2\, t^2\, u_2\, u_4^3 + 472\, u_5^4\, u_8\, t\, u_2^2\, u_4 \\
& + 290\, u_5^2\, u_6^2\, u_7\, t\, u_2^2\, u_3 + 54\, u_5\, u_6^2\, u_8\, t^2\, u_3\, u_4^2 - 52\, u_5^3\, u_7^2\, t^2\, u_3\, u_4 \\
& + 8\, u_6\, u_8^2\, t\, u_2^2\, u_3^2\, u_4 + 8\, u_7\, u_8^2\, u_5\, t^2\, u_2\, u_3^2 + 8\, u_7\, u_8^2\, u_5\, u_2^4\, t \\
& - 730\, u_7\, u_8\, u_2^2\, u_3^3\, u_4^2 + 66\, u_5\, u_7\, u_8\, t\, u_3^4\, u_4 - 102\, u_6^3\, u_5^2\, t\, u_2\, u_3^2 \\
& - 374\, u_6^2\, u_7\, t\, u_3^3\, u_4^2 + 8\, u_6\, u_8^2\, t\, u_2^3\, u_4^2 - 4\, u_5^3\, u_6\, u_8\, u_7\, t^3 + 110\, u_8^2\, u_2\, u_3^2\, u_4^3\, t \\
& + 200\, u_4^9 + u_8^3\, t^2\, u_2^2\, u_4^2 + u_6^2\, u_8^2\, t^3\, u_4^2 - 180\, u_8\, u_4^5\, u_3^2\, u_2 + 730\, u_7\, u_4^6\, u_2\, u_3 \\
& - 4020\, u_5\, u_6\, u_3^3\, u_4^4 - 590\, u_5\, u_7\, u_4^5\, u_2^2 + 1560\, u_5\, u_7\, u_3^4\, u_4^3 - 240\, u_5\, u_8\, u_3^5\, u_4^2 \\
& + 90\, u_8^2\, u_2^3\, u_3^2\, u_4^2 - 90\, u_8^2\, u_2^2\, u_3^4\, u_4 - 40\, u_8^2\, t\, u_2^2\, u_4^4 - 35\, u_8^2\, t\, u_3^4\, u_4^2 \\
& + 240\, u_7^2\, t\, u_2\, u_4^5 + 210\, u_6\, u_8\, u_3^6\, u_4 - 1020\, u_6\, u_7\, u_3^5\, u_4^2 - 1990\, u_6^2\, u_2\, u_3^2\, u_4^4 \\
& - 10\, u_5\, u_7\, u_4^6\, t + 790\, u_7^2\, u_2^2\, u_3^2\, u_4^3 - 130\, u_7^2\, t\, u_3^2\, u_4^4 - 860\, u_7^2\, u_2\, u_3^4\, u_4^2 \\
& - 200\, u_6^3\, t\, u_2\, u_4^4 + 2\, u_8^3\, t\, u_2^3\, u_3^2 - 52\, u_7^3\, t^2\, u_3\, u_4^3 - 2\, u_8^3\, t\, u_2^4\, u_4 \\
& - 4020\, u_5^3\, u_2\, u_3\, u_4^4 - 70\, u_7^3\, u_2^4\, u_3\, u_4 - 8\, u_7\, u_8^2\, u_2^5\, u_3 + 624\, u_5\, u_6\, u_7\, u_3^6 \\
& + 192\, u_5^2\, u_8\, u_2^3\, u_4^3 + 6\, u_6^2\, u_8\, u_2^4\, u_4^2 + 70\, u_6\, u_7^2\, u_2^4\, u_4^2 + 20\, u_7^2\, u_8\, u_2^5\, u_4 \\
& + 8\, u_7^2\, u_8\, u_2^4\, u_3^2 + 3900\, u_5^2\, u_6\, u_3^4\, u_4^2 - 1990\, u_5^2\, u_6\, u_4^4\, u_2^2 - 180\, u_5^2\, u_6\, u_4^5\, t \\
& + 28\, u_6\, u_8^2\, u_2^4\, u_3^2 - 28\, u_6\, u_8^2\, u_2^5\, u_4 + 288\, u_6\, u_7^2\, u_2^2\, u_3^4 + 130\, u_6\, u_7^2\, t^2\, u_4^4 \\
& - 392\, u_6^2\, u_7\, u_3^5\, u_2 + 256\, u_6^2\, u_8\, u_2^2\, u_3^4 - 40\, u_6^2\, u_8\, t^2\, u_4^4 - 368\, u_5\, u_7^2\, u_3^5\, u_2 \\
& + 72\, u_5\, u_8^2\, u_2^3\, u_3^3 + 58\, u_5\, u_8^2\, u_3^5\, t - 1452\, u_5\, u_6^2\, u_3^5\, u_4 - 1068\, u_5^2\, u_7\, u_3^5\, u_4 +
\end{aligned}
$$

$$
\begin{aligned}
&+6\,u_7{}^4\,t^2\,u_2\,u_3{}^2+6\,u_6{}^4\,t\,u_2{}^2\,u_4{}^2+3900\,u_5{}^4\,u_2\,u_3{}^2\,u_4{}^2-18\,u_7{}^3\,u_6\,t^2\,u_3{}^3\\
&-92\,u_5{}^3\,u_8\,u_2{}^2\,u_3{}^3-70\,u_6\,u_7\,t\,u_3\,u_4{}^5-244\,u_6\,u_8\,t\,u_2\,u_4{}^5+540\,u_5\,u_8\,u_2\,u_3{}^3\,u_4{}^3\\
&-240\,u_5\,u_8\,u_2{}^2\,u_3\,u_4{}^4+550\,u_7\,u_8\,u_2\,u_3{}^5\,u_4+40\,u_7\,u_8\,t\,u_3{}^3\,u_4{}^3+220\,u_6\,u_8\,t\,u_3{}^2\,u_4{}^4\\
&-760\,u_6\,u_8\,u_2\,u_3{}^4\,u_4{}^2-8\,u_7{}^2\,u_8\,t^2\,u_2\,u_4{}^3-390\,u_5\,u_6\,u_7\,t\,u_2\,u_4{}^4\\
&-2826\,u_5\,u_6\,u_7\,u_2{}^2\,u_3{}^2\,u_4{}^2+1510\,u_5\,u_6\,u_7\,u_2{}^3\,u_4{}^3+398\,u_5\,u_6\,u_7\,t\,u_3{}^2\,u_4{}^3\\
&+618\,u_5\,u_7\,u_8\,u_2{}^3\,u_3{}^2\,u_4-770\,u_5\,u_6\,u_8\,t\,u_3{}^3\,u_4{}^2+22\,u_6\,u_7\,u_8\,t^2\,u_3\,u_4{}^3\\
&+346\,u_6\,u_7\,u_8\,u_2{}^4\,u_3\,u_4+216\,u_5\,u_7\,u_8\,t\,u_2{}^2\,u_4{}^3+8\,u_7\,u_8{}^2\,t^2\,u_2\,u_4{}^2\,u_3\\
&-310\,u_5\,u_6\,u_8\,u_3{}^5\,u_2+972\,u_5\,u_6\,u_8\,t\,u_2\,u_4{}^3\,u_3-2\,u_8{}^3\,t^2\,u_2\,u_3{}^2\,u_4\\
&-194\,u_5\,u_7\,u_8\,u_2{}^2\,u_3{}^4-70\,u_5\,u_7\,u_8\,t^2\,u_4{}^4-216\,u_6\,u_7\,u_8\,u_2{}^3\,u_3{}^3\\
&+700\,u_6\,u_7\,u_8\,t\,u_2\,u_3{}^3\,u_4-512\,u_6\,u_7\,u_8\,t\,u_2{}^2\,u_4{}^2\,u_3-138\,u_5\,u_7\,u_8\,u_2\,u_3{}^2\,u_4{}^2\,t\\
&+284\,u_5\,u_7{}^2\,t\,u_3{}^3\,u_4{}^2-240\,u_5\,u_6{}^2\,t\,u_3\,u_4{}^4+1336\,u_5\,u_7{}^2\,u_2{}^2\,u_3{}^3\,u_4\\
&-1060\,u_5\,u_7{}^2\,u_2{}^3\,u_3\,u_4{}^2-418\,u_5\,u_6{}^2\,u_2{}^2\,u_3\,u_4{}^3-630\,u_5{}^2\,u_8\,u_2{}^2\,u_3{}^2\,u_4{}^2\\
&+168\,u_5{}^2\,u_8\,u_2\,u_3{}^4\,u_4+300\,u_5{}^2\,u_8\,t\,u_3{}^2\,u_4{}^3+220\,u_5{}^2\,u_8\,t\,u_2\,u_4{}^4\\
&+3898\,u_5{}^2\,u_7\,u_2{}^2\,u_3\,u_4{}^3+120\,u_5{}^2\,u_7\,t\,u_3\,u_4{}^4+20\,u_6\,u_8{}^2\,t^2\,u_3{}^2\,u_4{}^2\\
&+834\,u_6\,u_7{}^2\,u_2\,u_3{}^2\,u_4{}^2\,t+54\,u_5\,u_8{}^2\,t\,u_2{}^2\,u_4{}^2\,u_3-496\,u_5\,u_7{}^2\,t\,u_2\,u_4{}^3\,u_3\\
&+10\,u_6\,u_8{}^2\,t\,u_2\,u_3{}^4-218\,u_6\,u_7{}^2\,t\,u_3{}^4\,u_4+524\,u_6{}^2\,u_8\,t\,u_2{}^2\,u_4{}^3+472\,u_6{}^2\,u_8\,t\,u_3{}^4\,u_4\\
&-50\,u_6{}^2\,u_7\,u_2{}^2\,u_3{}^3\,u_4-334\,u_5\,u_6\,u_8\,u_7\,t\,u_2{}^2\,u_3{}^2-130\,u_5\,u_6\,u_8\,u_7\,t^2\,u_3{}^2\,u_4\\
&-174\,u_5\,u_6\,u_8\,u_7\,u_2{}^5+62\,u_7{}^3\,u_5\,t^2\,u_2\,u_4{}^2+1020\,u_5{}^2\,u_6\,u_8\,t\,u_2\,u_3{}^2\,u_4\\
&-102\,u_5{}^2\,u_6\,u_8\,u_2{}^3\,u_3{}^2+350\,u_5{}^2\,u_6\,u_8\,u_2{}^4\,u_4-50\,u_5{}^2\,u_6\,u_8\,u_3{}^4\,t\\
&-2\,u_7{}^3\,u_6\,t^2\,u_2\,u_3\,u_4-28\,u_6{}^3\,u_5\,u_2{}^3\,u_3\,u_4+360\,u_6{}^3\,u_5\,t\,u_3{}^3\,u_4\\
&+260\,u_5\,u_6\,u_8\,u_7\,t\,u_2{}^3\,u_4+18\,u_5\,u_6\,u_8\,u_7\,t^2\,u_2\,u_4{}^2-330\,u_5{}^3\,u_7\,t\,u_3{}^2\,u_4{}^2\\
&-4710\,u_5{}^3\,u_6\,u_2\,u_3{}^3\,u_4-250\,u_5{}^3\,u_6\,u_2{}^2\,u_3\,u_4{}^2+540\,u_5{}^3\,u_6\,t\,u_3\,u_4{}^3\\
&-1126\,u_5{}^2\,u_6\,u_8\,t\,u_2{}^2\,u_4{}^2-104\,u_6{}^2\,u_7{}^2\,t\,u_2{}^2\,u_3{}^2+102\,u_5{}^2\,u_8{}^2\,t\,u_2{}^2\,u_3{}^2\\
&+30\,u_5{}^2\,u_8{}^2\,t^2\,u_3{}^2\,u_4+298\,u_5{}^2\,u_7{}^2\,t\,u_2{}^2\,u_4{}^2-32\,u_6{}^3\,u_8\,t^2\,u_3{}^2\,u_4+8\,u_6{}^3\,u_8\,t^2\,u_2\,u_4{}^2\\
&+116\,u_6{}^3\,u_8\,t\,u_2{}^2\,u_3{}^2+94\,u_6{}^3\,u_7\,t\,u_2\,u_3{}^3+674\,u_5\,u_6{}^2\,u_8\,t\,u_2{}^2\,u_3\,u_4\\
&+20\,u_5{}^2\,u_8{}^2\,t^2\,u_2\,u_4{}^2-654\,u_6{}^3\,u_5\,t\,u_2\,u_4{}^2\,u_3-770\,u_5{}^3\,u_8\,t\,u_2\,u_4{}^2\,u_3\\
&-26\,u_5\,u_6{}^2\,u_8\,u_2{}^4\,u_3-1330\,u_6{}^3\,u_5\,u_2{}^2\,u_3{}^3-1486\,u_5{}^3\,u_7\,u_2{}^3\,u_4{}^2+852\,u_5{}^3\,u_7\,u_3{}^4\,u_2\\
&-304\,u_6{}^3\,u_7\,u_2{}^4\,u_3+386\,u_5{}^2\,u_6{}^2\,u_2{}^3\,u_4{}^2+1598\,u_5{}^2\,u_6{}^2\,u_3{}^4\,u_2-514\,u_5{}^2\,u_7{}^2\,u_2{}^3\,u_3{}^2\\
&+460\,u_5{}^2\,u_7{}^2\,u_2{}^4\,u_4-142\,u_5{}^2\,u_7{}^2\,u_3{}^4\,t+9\,u_6{}^2\,u_8{}^2\,u_2{}^4\,t-200\,u_6\,u_8\,u_2{}^3\,u_4{}^4\\
&-240\,u_5{}^5\,t\,u_3\,u_4{}^2-1452\,u_5{}^5\,u_2{}^2\,u_3\,u_4-28\,u_6{}^5\,t^2\,u_2\,u_4+724\,u_5{}^4\,u_7\,u_2{}^3\,u_3\\
&-60\,u_5{}^4\,u_7\,u_3{}^3\,t-1330\,u_5{}^3\,u_6{}^2\,u_2{}^3\,u_3-92\,u_5{}^3\,u_6{}^2\,u_3{}^3\,t+9\,u_6{}^4\,u_8\,t^2\,u_2{}^2\\
&-2\,u_6{}^4\,u_8\,t^3\,u_4-150\,u_5{}^3\,u_6\,u_7\,u_2{}^4-35\,u_5{}^4\,u_8\,t^2\,u_4{}^2+386\,u_6{}^3\,u_2{}^2\,u_3{}^2\,u_4{}^2\\
&+326\,u_6{}^3\,u_2\,u_3{}^4\,u_4+192\,u_6{}^3\,t\,u_3{}^2\,u_4{}^3+90\,u_6{}^3\,u_5{}^2\,t^2\,u_4{}^2+2\,u_6{}^3\,u_7{}^2\,t^3\,u_4\\
&-90\,u_5{}^4\,u_6{}^2\,t^2\,u_4+256\,u_5{}^4\,u_6{}^2\,t\,u_2{}^2+72\,u_5{}^3\,u_6{}^3\,t^2\,u_3+70\,u_5{}^4\,u_7{}^2\,t^2\,u_2\\
&+4\,u_6{}^2\,u_7{}^2\,u_5{}^2\,t^3+28\,u_6{}^4\,u_5{}^2\,t^2\,u_2-10\,u_5{}^5\,u_7\,t^2\,u_4-250\,u_5{}^5\,u_7\,t\,u_2{}^2\\
&+58\,u_5{}^5\,u_8\,t^2\,u_3+2\,u_6{}^3\,u_5{}^2\,u_8\,t^3-4\,u_6{}^4\,u_5\,u_7\,t^3+1598\,u_5{}^4\,u_6\,u_2{}^2\,u_3{}^2\\
&+326\,u_5{}^4\,u_6\,u_2{}^3\,u_4+210\,u_5{}^6\,t\,u_2\,u_4+20\,u_5{}^2\,u_7\,u_8\,t\,u_2\,u_3{}^3-630\,u_5{}^2\,u_6{}^2\,t\,u_3{}^2\,u_4{}^2\\
&+682\,u_5\,u_6{}^2\,u_7\,t\,u_2{}^2\,u_4{}^2+880\,u_5{}^2\,u_6{}^2\,t\,u_2\,u_4{}^3+2010\,u_5{}^2\,u_6{}^2\,u_2{}^2\,u_3{}^2\,u_4\\
&-878\,u_5\,u_6{}^2\,u_7\,t\,u_2\,u_3{}^2\,u_4-290\,u_5\,u_6{}^2\,u_7\,t^2\,u_4{}^3+766\,u_5\,u_6{}^2\,u_7\,u_2{}^3\,u_3{}^2\\
&-772\,u_5\,u_6{}^2\,u_7\,u_2{}^4\,u_4+236\,u_5\,u_6{}^2\,u_7\,u_3{}^4\,t+34\,u_5{}^2\,u_7\,u_8\,t\,u_2{}^2\,u_3\,u_4\\
&+18\,u_5{}^2\,u_7{}^2\,t\,u_2\,u_3{}^2\,u_4-456\,u_5{}^2\,u_6\,u_7\,t\,u_2\,u_4{}^2\,u_3-10\,u_5\,u_6\,u_8{}^2\,t^2\,u_2\,u_3\,u_4\\
&-10\,u_5\,u_6\,u_8{}^2\,t^2\,u_3{}^3-178\,u_6{}^3\,u_7\,t\,u_2{}^2\,u_3\,u_4-12\,u_7{}^2\,u_8\,u_6\,t^2\,u_2\,u_3{}^2\\
&-8\,u_7\,u_8{}^2\,u_5\,t^2\,u_2{}^2\,u_4-184\,u_5\,u_6\,u_7{}^2\,t^2\,u_3\,u_4{}^2+130\,u_5\,u_6\,u_7{}^2\,t\,u_2\,u_3{}^3\\
&-30\,u_7{}^2\,u_8\,u_6\,u_2{}^4\,t-2\,u_7{}^2\,u_8\,u_6\,t^3\,u_4{}^2+2\,u_7{}^2\,u_8\,u_5\,t^2\,u_3{}^3-14\,u_7{}^2\,u_8\,u_5\,t^2\,u_2\,u_3\,u_4\\
&-366\,u_5\,u_6\,u_7{}^2\,t\,u_2{}^2\,u_3\,u_4+30\,u_6{}^2\,u_7\,u_8\,t^2\,u_3{}^3+14\,u_6{}^2\,u_7\,u_8\,t^2\,u_2\,u_3\,u_4\\
&-26\,u_6{}^4\,u_5\,t\,u_2{}^2\,u_3+354\,u_5{}^2\,u_6{}^2\,u_7\,t^2\,u_3\,u_4+534\,u_5{}^4\,u_7\,u_2\,u_3\,u_4\,t\\
&-50\,u_5{}^4\,u_8\,t\,u_2\,u_3{}^2-10\,u_5{}^3\,u_8{}^2\,t^2\,u_2\,u_3+734\,u_5{}^3\,u_6{}^2\,u_2\,u_3\,u_4\,t-48\,u_5{}^3\,u_7{}^2\,t\,u_2{}^2\,u_3\\
&+74\,u_5{}^2\,u_6\,u_8\,u_7\,t^2\,u_2\,u_3+88\,u_6{}^2\,u_7{}^2\,u_5\,t^2\,u_2\,u_3+118\,u_6{}^3\,u_5\,u_7\,t^2\,u_2\,u_4+
\end{aligned}
$$

$$+ 28\,u_5{}^3\,u_6\,u_7\,t\,u_2\,u_3{}^2 - 54\,u_5{}^3\,u_7\,u_8\,t^2\,u_2\,u_4 - 304\,u_5{}^3\,u_6\,u_8\,t\,u_2{}^2\,u_3$$
$$- 206\,u_5{}^3\,u_6\,u_8\,t^2\,u_3\,u_4 + 8\,u_5{}^2\,u_6{}^2\,u_8\,t^2\,u_2\,u_4 - 24\,u_5\,u_6{}^2\,u_8\,u_7\,t^2\,u_2{}^2$$
$$+ 4\,u_5\,u_6{}^2\,u_8\,u_7\,t^3\,u_4 - 208\,u_6{}^3\,u_5\,u_7\,t^2\,u_3{}^2 - 254\,u_6{}^3\,u_5\,u_7\,u_2{}^3\,t$$
$$+ 130\,u_5{}^3\,u_6\,u_7\,t^2\,u_4{}^2 + 104\,u_5{}^2\,u_7{}^2\,u_6\,t^2\,u_3{}^2 + 170\,u_5{}^2\,u_7{}^2\,u_6\,u_2{}^3\,t - 4\,u_7{}^3\,u_5\,u_6\,t^3\,u_4$$
$$- 86\,u_5{}^2\,u_7{}^2\,u_6\,t^2\,u_2\,u_4 + 102\,u_5{}^2\,u_6{}^2\,u_8\,t^2\,u_3{}^2 + 116\,u_5{}^2\,u_6{}^2\,u_8\,u_2{}^3\,t$$
$$- 46\,u_6{}^3\,u_5\,u_8\,t^2\,u_2\,u_3 - 260\,u_7{}^3\,u_5\,t\,u_2{}^3\,u_4 + 10\,u_5{}^4\,u_8\,u_6\,t^2\,u_2.$$

$$f_{10} = 380020\,u_8\,u_4{}^7\,u_2{}^2 + 1876355\,u_6{}^{????}\,u_4{}^6\,u_2{}^2 + 179985\,u_8{}^2\,u_2{}^4\,u_4{}^4 + 2056620\,u_5{}^2\,u_4{}^6\,u_3{}^2$$
$$- 100320\,u_5{}^3\,u_7\,u_3{}^6 + 184950\,u_6{}^2\,u_8{}^2\,u_2{}^6 + 690060\,u_6\,u_4{}^7\,u_3{}^2 - 300\,u_8{}^3\,u_2{}^5\,u_3{}^2$$
$$+ 625992\,u_6{}^4\,u_2{}^4\,u_4{}^2 + 733515\,u_5{}^4\,u_3{}^4\,u_4{}^2 + 738120\,u_5\,u_7{}^2\,u_8\,u_2{}^5\,u_3$$
$$+ 1498560\,u_5\,u_6\,u_8\,u_7\,u_2{}^5\,u_4 + 2245300\,u_5\,u_6\,u_4{}^6\,u_2\,u_3 - 1848780\,u_6{}^3\,u_2{}^3\,u_4{}^4$$
$$+ 358020\,u_8\,u_4{}^5\,u_3{}^4 + 491610\,u_5{}^6\,u_2{}^2\,u_3{}^2 + 690060\,u_5{}^2\,u_4{}^7\,u_2$$
$$+ 1037715\,u_5{}^4\,u_2{}^2\,u_4{}^4 - 772080\,u_5\,u_4{}^8\,u_3 + 491610\,u_5{}^2\,u_6{}^2\,u_3{}^6 + 380020\,u_6{}^2\,u_4{}^7\,t$$
$$- 294\,u_5{}^2\,u_7{}^2\,u_6\,u_8\,t^3\,u_2 - 2087040\,u_5{}^3\,u_4{}^4\,u_3{}^3 + 183366\,u_7{}^4\,u_3{}^4\,t^2$$
$$+ 1037715\,u_6{}^2\,u_4{}^4\,u_3{}^4 + 270\,u_7{}^4\,u_4{}^3\,t^3 + 184950\,u_6{}^6\,t^2\,u_2{}^2 + 174600\,u_6{}^2\,u_7\,u_3{}^7$$
$$+ 35400\,u_7{}^3\,u_2{}^2\,u_3{}^5 + 358020\,u_5{}^4\,u_4{}^5\,t + 450\,u_6{}^6\,t^3\,u_4 - 320\,u_5\,u_6{}^2\,u_7{}^3\,t^3\,u_2$$
$$- 305550\,u_6{}^3\,u_3{}^6\,u_4 + 2235376\,u_6{}^4\,u_5\,t\,u_2{}^2\,u_3\,u_4 - 1116690\,u_6{}^2\,u_7\,u_3{}^5\,u_2\,u_4$$
$$- 3632765\,u_5{}^4\,u_8\,u_3{}^2\,t\,u_4\,u_2 - 5455\,u_5\,u_6{}^2\,u_8\,u_3\,t^2\,u_4{}^3 + 730757\,u_7{}^3\,u_6\,t^2\,u_2\,u_4{}^2\,u_3$$
$$+ 450\,u_8{}^3\,u_2{}^6\,u_4 + 383020\,u_7{}^2\,u_2{}^3\,u_4{}^5 - 764060\,u_6\,u_4{}^8\,u_2 + 184910\,u_8{}^2\,u_4{}^6\,t^2$$
$$- 611040\,u_7\,u_4{}^6\,u_3{}^3 + 1471780\,u_5{}^5\,u_6\,u_3\,t\,u_4\,u_2 - 3500427\,u_5{}^2\,u_8\,u_6\,u_2{}^3\,u_3{}^2\,u_4$$
$$- 163\,u_7{}^3\,u_5\,u_8\,t^3\,u_2\,u_4 + 1791630\,u_6\,u_7{}^2\,u_2{}^2\,u_3{}^4\,u_4 - 305550\,u_5{}^6\,u_2{}^3\,u_4$$
$$+ 24000\,u_5\,u_7{}^2\,u_3{}^7 - 334\,u_6\,u_8{}^2\,u_7\,u_5\,u_3{}^2\,t^3 + 123390\,u_5{}^5\,u_7\,u_2{}^4$$
$$- 1062\,u_5{}^2\,u_8{}^2\,u_7\,t^2\,u_2{}^2\,u_3 + 1108960\,u_6\,u_8{}^2\,u_2{}^4\,u_3{}^2\,u_4 + 351\,u_6{}^2\,u_7{}^2\,u_8\,t^3\,u_4\,u_2$$
$$- 5007155\,u_5{}^2\,u_8\,u_2{}^2\,u_4{}^3\,u_3{}^2 + 1985866\,u_5{}^3\,u_6\,u_7\,u_2{}^3\,u_3{}^2 + 368370\,u_6\,u_8{}^2\,u_3{}^2\,t^2\,u_4{}^3$$
$$+ 1433776\,u_5{}^3\,u_6{}^3\,t\,u_2{}^2\,u_3 - 300\,u_6{}^5\,u_5{}^2\,t^3 + 179985\,u_6{}^4\,u_4{}^4\,t^2 + 731040\,u_7{}^2\,u_3{}^6\,u_4{}^2$$
$$- 4420080\,u_5\,u_7\,u_4{}^5\,u_3{}^2\,u_2 + 26\,u_7{}^2\,u_8{}^2\,u_6\,t^3\,u_2{}^2 - 49\,u_5\,u_7{}^2\,u_8{}^2\,t^3\,u_2\,u_3$$
$$- 4000620\,u_5{}^2\,u_8\,u_3{}^2\,u_4{}^4\,t + 350\,u_5{}^2\,u_8{}^2\,u_6\,u_4{}^2\,t^3 - 737540\,u_6\,u_7\,u_4{}^6\,u_3\,t$$
$$- 363020\,u_8\,u_4{}^8\,t - 236\,u_7{}^3\,u_8\,u_3\,t^3\,u_4{}^2 + 735980\,u_5{}^4\,u_8\,u_7\,t^2\,u_2\,u_3$$
$$+ 4502365\,u_6\,u_8\,u_2{}^2\,u_4{}^4\,u_3{}^2 - 3609600\,u_5{}^2\,u_7\,u_4{}^5\,t\,u_3 + 370\,u_5{}^3\,u_7{}^2\,u_8\,t^3\,u_3$$
$$+ 2108315\,u_5\,u_7{}^2\,u_6\,u_2{}^4\,u_3\,u_4 - 734254\,u_5\,u_6\,u_8{}^2\,t\,u_2{}^2\,u_3{}^3 - 365985\,u_6{}^3\,u_7{}^2\,t^2\,u_2\,u_3{}^2$$
$$+ 737450\,u_5{}^2\,u_6{}^2\,u_7{}^2\,t^2\,u_2{}^2 + 5804400\,u_5\,u_7{}^2\,u_2{}^2\,u_3{}^3\,u_4{}^2 + 2388072\,u_5{}^4\,u_7\,u_2{}^3\,u_3\,u_4$$
$$- 200\,u_5{}^2\,u_7{}^2\,u_8\,u_4{}^2\,t^3 + 1702690\,u_5\,u_6\,u_7\,u_3{}^4\,u_4{}^2\,u_2 - 2152422\,u_5{}^3\,u_7{}^2\,t\,u_2\,u_3{}^3$$
$$- 1165\,u_7{}^3\,u_5\,u_6\,u_4{}^2\,t^3 + 182760\,u_7{}^4\,u_2{}^6 + 1868794\,u_5\,u_6{}^2\,u_7\,u_2{}^2\,u_3{}^4$$
$$+ 763815\,u_5\,u_6{}^2\,u_7\,u_4{}^4\,t^2 - 741210\,u_7{}^3\,u_6\,t\,u_2{}^2\,u_3{}^3 + 3592225\,u_5\,u_6\,u_8\,u_2{}^2\,u_3{}^3\,u_4{}^2$$
$$+ 2178820\,u_5\,u_8\,u_4{}^6\,u_3\,t - 950\,u_5{}^3\,u_8\,u_6\,u_7\,t^3\,u_4 - 2899540\,u_5\,u_6{}^2\,u_7\,u_2{}^4\,u_4{}^2$$
$$- 5141945\,u_6\,u_7{}^2\,u_3{}^2\,t\,u_4{}^3\,u_2 + 10\,u_8{}^3\,u_6\,u_3{}^2\,t^3\,u_4 + 372712\,u_5{}^2\,u_8{}^2\,u_6\,t^2\,u_2\,u_3{}^2$$
$$+ 335615\,u_5{}^2\,u_6{}^2\,u_2\,u_3{}^4\,u_4 - 364760\,u_5{}^4\,u_8\,u_6\,u_3{}^2\,t^2 + 8237355\,u_5\,u_7\,u_8\,u_2{}^3\,u_4{}^2\,u_3{}^2$$
$$+ 2969648\,u_6{}^3\,u_5\,u_7\,t\,u_2{}^3\,u_4 + 739293\,u_5{}^2\,u_8\,u_6\,u_7\,u_3{}^3\,t^2 + 729160\,u_6{}^2\,u_7\,u_8\,u_2{}^5\,u_3$$
$$+ 684160\,u_5{}^3\,u_6{}^2\,u_7\,u_2{}^3\,t - 368122\,u_6\,u_7{}^2\,u_8\,t^2\,u_2{}^2\,u_4{}^2 + 5015205\,u_5{}^3\,u_7\,u_3{}^2\,t\,u_4{}^3$$
$$+ 77100\,u_5{}^5\,u_3{}^5 + 360\,u_5\,u_6{}^3\,u_7{}^2\,t^3\,u_3 + 740862\,u_6{}^4\,u_7\,t^2\,u_2\,u_4\,u_3$$
$$- 7606100\,u_5\,u_7{}^2\,u_2{}^3\,u_4{}^3\,u_3 + 3592225\,u_5{}^3\,u_6{}^2\,u_3\,t\,u_4{}^2\,u_2 + 4326150\,u_5{}^3\,u_6{}^2\,u_3{}^3\,t\,u_4$$
$$- 727344\,u_6{}^3\,u_8\,t\,u_2\,u_3{}^4 - 372670\,u_5{}^4\,u_7{}^2\,t^2\,u_2\,u_4 + 2215290\,u_7{}^3\,u_3{}^3\,t\,u_4{}^2\,u_2$$
$$+ 1481289\,u_5{}^3\,u_8\,u_6\,t\,u_2\,u_3{}^3 - 130\,u_8{}^3\,u_5\,t^2\,u_2\,u_3{}^3 + 2197275\,u_5\,u_7\,u_8\,u_2{}^2\,u_4{}^4\,t$$
$$+ 4380240\,u_5\,u_8\,u_4{}^4\,u_3{}^3\,u_2 - 734229\,u_6{}^3\,u_5\,u_8\,u_3{}^3\,t^2 + 49\,u_8{}^3\,u_5\,u_6\,t^3\,u_2\,u_3$$
$$+ 1462770\,u_5{}^5\,u_8\,t\,u_2{}^2\,u_3 - 2164520\,u_5{}^4\,u_7\,u_3\,t\,u_4{}^2\,u_2 + 373023\,u_5{}^2\,u_7{}^2\,u_6\,u_3{}^2\,t^2\,u_4$$
$$- 705590\,u_5\,u_7\,u_8\,u_3{}^4\,u_4{}^2\,t - 2196645\,u_5{}^4\,u_6\,u_3{}^2\,t\,u_4{}^2$$
$$- 2423288\,u_5{}^2\,u_6\,u_7\,u_2{}^2\,u_3{}^3\,u_4 - 67\,u_7{}^3\,u_6\,u_8\,t^3\,u_2\,u_3 - 1973035\,u_6{}^2\,u_7\,u_2{}^2\,u_3{}^3\,u_4{}^2$$
$$+ 683020\,u_5{}^2\,u_8\,u_6\,u_2{}^2\,u_4{}^3\,t - 2115175\,u_5\,u_6{}^2\,u_7\,u_2{}^2\,u_4{}^3\,t -$$

$$
\begin{aligned}
&- 734254\, u_5{}^3\, u_6{}^2\, u_8\, t^2\, u_2\, u_3 - 3093177\, u_5{}^2\, u_8\, u_7\, u_2{}^4\, u_3\, u_4 \\
&+ 1313\, u_7\, u_8{}^2\, u_5\, t^2\, u_2{}^2\, u_4{}^2 + 334\, u_7{}^3\, u_5\, u_8\, u_3{}^2\, t^3 - 1457739\, u_6{}^3\, u_5\, u_7\, u_3{}^2\, t^2\, u_4 \\
&+ 4322715\, u_5{}^3\, u_8\, u_3\, t\, u_4{}^3\, u_2 - 738540\, u_5\, u_6\, u_8{}^2\, u_2{}^5\, u_3 - 703440\, u_5{}^4\, u_8\, t\, u_2{}^2\, u_4{}^2 \\
&+ 175\, u_5\, u_6{}^2\, u_8{}^2\, u_3\, t^3\, u_4 - 818240\, u_5\, u_8\, u_4{}^5\, u_2{}^2\, u_3 + 4322715\, u_5\, u_6\, u_8\, u_3{}^3\, u_4{}^3\, t \\
&- 2883690\, u_5\, u_7{}^2\, u_3{}^3\, u_4{}^3\, t + 1180\, u_7\, u_8{}^2\, u_5\, u_3{}^4\, t^2 - 1481230\, u_5{}^3\, u_7{}^2\, u_3\, t^2\, u_4{}^2 \\
&- 1237\, u_7{}^3\, u_8\, t^2\, u_2{}^2\, u_4\, u_3 - 2026175\, u_6\, u_7{}^2\, u_2{}^3\, u_4{}^2\, u_3{}^2 + 366130\, u_7{}^2\, u_8\, u_4{}^4\, t^2\, u_2 \\
&- 1445980\, u_5{}^2\, u_8\, u_4{}^5\, t\, u_2 - 703440\, u_6{}^2\, u_8\, u_3{}^4\, u_4{}^2\, t + 436\, u_7{}^4\, u_5\, t^2\, u_2{}^2\, u_3 \\
&- 566\, u_5{}^2\, u_7{}^3\, u_6\, t^3\, u_3 - 1759\, u_5\, u_6{}^2\, u_8{}^2\, t^2\, u_2{}^2\, u_3 + 733440\, u_5{}^5\, u_6\, u_7\, t^2\, u_2 \\
&+ 186510\, u_5{}^8\, t^2 - 369358\, u_6{}^4\, u_8\, t^2\, u_2{}^2\, u_4 - 2210200\, u_5\, u_6\, u_8\, u_7\, u_2{}^4\, u_3{}^2 \\
&+ 368370\, u_5{}^2\, u_8{}^2\, t^2\, u_2\, u_4{}^3 + 10944\, u_6{}^5\, u_2{}^5 - 2241245\, u_5{}^3\, u_6\, u_7\, u_4{}^3\, t^2 \\
&- 2189620\, u_5{}^3\, u_6{}^2\, u_7\, t^2\, u_2\, u_4 + 1101435\, u_6{}^2\, u_7{}^2\, u_3{}^2\, t^2\, u_4{}^2 \\
&+ 1470780\, u_5{}^2\, u_8\, u_6\, u_7\, t^2\, u_2\, u_4\, u_3 + 865\, u_5\, u_6{}^2\, u_7\, u_8\, u_4{}^2\, t^3 - 746265\, u_6{}^3\, u_7\, u_3\, t^2\, u_4{}^3 \\
&+ 2929155\, u_5{}^2\, u_6{}^2\, u_7\, u_3\, t^2\, u_4{}^2 + 366993\, u_5{}^2\, u_8{}^2\, u_6\, u_2{}^4\, t + 731240\, u_5\, u_8{}^2\, u_3{}^5\, t\, u_4 \\
&+ 1493005\, u_7\, u_8\, u_2{}^3\, u_4{}^4\, u_3 - 94\, u_5{}^2\, u_6{}^2\, u_7\, u_8\, t^3\, u_3 - 5246655\, u_7\, u_8\, u_3{}^3\, u_4{}^3\, u_2{}^2 \\
&+ 1108960\, u_6{}^4\, u_5{}^2\, t^2\, u_2\, u_4 - 366980\, u_6\, u_8{}^2\, u_4{}^4\, t^2\, u_2 + 2132620\, u_5{}^5\, u_7\, t\, u_2\, u_3{}^2 \\
&+ 56\, u_7{}^2\, u_8{}^2\, t^3\, u_2\, u_4{}^2 + 368862\, u_6{}^2\, u_8{}^2\, t\, u_2{}^3\, u_3{}^2 - 1453480\, u_7\, u_8\, u_3\, t\, u_4{}^5\, u_2 \\
&+ 1502980\, u_5{}^4\, u_6\, u_7\, u_3\, t^2\, u_4 + 728768\, u_7{}^3\, u_5\, t\, u_2\, u_3{}^4 - 738540\, u_6{}^5\, u_5\, t^2\, u_2\, u_3 \\
&+ 1431986\, u_5{}^3\, u_6\, u_7\, u_3{}^4\, t + 2185745\, u_7{}^3\, u_5\, u_2{}^3\, u_4{}^2\, t - 849\, u_6{}^3\, u_7\, u_8\, u_3\, t^3\, u_4 \\
&+ 365629\, u_5{}^2\, u_7{}^2\, u_8\, t^2\, u_2{}^2\, u_4 + 737967\, u_5\, u_6{}^2\, u_7\, u_8\, t^2\, u_2{}^2\, u_4 \\
&+ 2567280\, u_5{}^2\, u_6{}^2\, u_8\, u_3{}^2\, t^2\, u_4 - 369358\, u_6{}^2\, u_8{}^2\, t\, u_2{}^4\, u_4 + 186510\, u_8{}^2\, u_3{}^8 \\
&+ 738420\, u_6\, u_8{}^2\, u_2{}^3\, u_4{}^3\, t - 369134\, u_5{}^2\, u_7{}^2\, u_8\, t^2\, u_2\, u_3{}^2 - 1457430\, u_5{}^5\, u_7\, t\, u_2{}^2\, u_4 \\
&+ 1800\, u_5{}^2\, u_6{}^2\, u_7{}^2\, t^3\, u_4 - 736942\, u_5{}^3\, u_8\, u_6\, u_7\, t^2\, u_2{}^2 + 1433776\, u_5\, u_6{}^2\, u_8\, u_2{}^3\, u_3{}^3 \\
&+ 763170\, u_6{}^4\, u_5\, u_3\, t^2\, u_4{}^2 + 1065235\, u_6\, u_7{}^2\, u_2{}^2\, u_4{}^4\, t + 5247690\, u_6{}^2\, u_7\, u_2{}^3\, u_4{}^3\, u_3 \\
&- 179\, u_6\, u_8{}^2\, u_7\, u_5\, t^2\, u_2{}^3 + 730026\, u_5\, u_6{}^2\, u_7{}^2\, u_3{}^3\, t^2 + 1297580\, u_7\, u_4{}^7\, u_3\, u_2 \\
&- 1091560\, u_8\, u_4{}^6\, u_3{}^2\, u_2 - 1432080\, u_5\, u_8\, u_3{}^5\, u_4{}^3 - 1954610\, u_6{}^2\, u_4{}^5\, u_3{}^2\, u_2 \\
&+ 2360160\, u_5\, u_7\, u_4{}^4\, u_3{}^4 - 1386580\, u_5\, u_7\, u_4{}^6\, u_2{}^2 + 722540\, u_5\, u_7\, u_4{}^7\, t \\
&- 3015410\, u_7{}^2\, u_4{}^3\, u_3{}^4\, u_2 - 337270\, u_7{}^2\, u_4{}^6\, u_2\, t + 349870\, u_7{}^2\, u_4{}^5\, u_3{}^2\, t \\
&- 746040\, u_7\, u_8\, u_3{}^7\, u_4 - 1115935\, u_8{}^2\, u_2{}^3\, u_4{}^3\, u_3{}^2 + 2051110\, u_8{}^2\, u_3{}^4\, u_4{}^2\, u_2{}^2 \\
&- 1489080\, u_6\, u_8\, u_2{}^3\, u_4{}^5 + 1095060\, u_6\, u_8\, u_3{}^6\, u_4{}^2 + 3071010\, u_7{}^2\, u_2{}^2\, u_4{}^4\, u_3{}^2 \\
&- 1919370\, u_6\, u_7\, u_3{}^5\, u_4{}^3 - 1119060\, u_8{}^2\, u_3{}^6\, u_2\, u_4 - 367220\, u_8{}^2\, u_4{}^5\, u_2{}^2\, t \\
&- 366770\, u_8{}^2\, u_4{}^3\, u_3{}^4\, t + 1844535\, u_6{}^3\, u_3{}^4\, u_4{}^2\, u_2 + 1138920\, u_6{}^3\, u_3{}^2\, u_4{}^4\, t \\
&- 100\, u_8{}^3\, u_3{}^4\, t^2\, u_4 - 844590\, u_7{}^3\, u_2{}^3\, u_3{}^3\, u_4 - 633570\, u_5\, u_6{}^2\, u_3{}^5\, u_4{}^2 \\
&- 1478970\, u_5{}^2\, u_7\, u_3{}^5\, u_4{}^2 + 1061460\, u_5{}^2\, u_8\, u_3{}^6\, u_4 - 707640\, u_5\, u_6\, u_8\, u_3{}^7 \\
&+ 1138920\, u_5{}^2\, u_8\, u_2{}^3\, u_4{}^4 + 225\, u_8{}^3\, u_2{}^4\, u_4{}^2\, t - 3008880\, u_5{}^3\, u_4{}^5\, u_3\, u_2 \\
&- 737640\, u_7{}^3\, u_3{}^5\, t\, u_4 + 1562955\, u_7{}^3\, u_2{}^4\, u_4{}^2\, u_3 + 219205\, u_6{}^3\, u_2{}^2\, u_4{}^3\, u_3{}^2 \\
&- 1489080\, u_6{}^3\, u_4{}^5\, t\, u_2 - 320\, u_8{}^3\, u_2{}^2\, u_4{}^3\, t^2 + 3740\, u_7{}^3\, u_4{}^4\, t^2\, u_3 \\
&+ 3120390\, u_5{}^2\, u_6\, u_4{}^3\, u_3{}^4 - 1091560\, u_5{}^2\, u_6\, u_4{}^6\, t - 1954610\, u_5{}^2\, u_6\, u_4{}^5\, u_2{}^2 \\
&+ 746840\, u_5\, u_8{}^2\, u_2{}^2\, u_3{}^5 + 2000\, u_7\, u_8{}^2\, u_2{}^4\, u_3{}^3 + 371820\, u_7{}^2\, u_8\, u_3{}^6\, t \\
&- 370820\, u_6\, u_8{}^2\, u_2{}^3\, u_3{}^4 - 363420\, u_6\, u_8{}^2\, u_3{}^6\, t + 350895\, u_7{}^2\, u_8\, u_2{}^5\, u_4{}^2 \\
&+ 355920\, u_7{}^2\, u_8\, u_2{}^3\, u_3{}^4 - 367770\, u_6\, u_8{}^2\, u_2{}^5\, u_4{}^2 - 367220\, u_6{}^2\, u_8\, u_4{}^5\, t^2 + 96510\, u_4{}^{10} \\
&+ 3743200\, u_7\, u_8\, u_3{}^5\, u_4{}^2\, u_2 + 1101310\, u_8{}^2\, u_3{}^2\, u_4{}^4\, t\, u_2 + 727540\, u_7\, u_8\, u_4{}^4\, u_3{}^3\, t \\
&+ 1443480\, u_6\, u_8\, u_4{}^6\, u_2\, t - 1445980\, u_6\, u_8\, u_4{}^5\, u_3{}^2\, t - 4439365\, u_6\, u_8\, u_4{}^3\, u_3{}^4\, u_2 \\
&- 4987905\, u_6\, u_7\, u_4{}^5\, u_2{}^2\, u_3 + 6125485\, u_6\, u_7\, u_4{}^4\, u_3{}^3\, u_2 - 2196645\, u_5{}^2\, u_8\, u_3{}^4\, u_4{}^2\, u_2 \\
&- 2850725\, u_5\, u_6\, u_8\, u_3\, t\, u_4{}^4\, u_2 + 4314010\, u_5\, u_7{}^2\, u_3\, t\, u_4{}^4\, u_2 - 818240\, u_5\, u_6{}^2\, u_4{}^5\, t\, u_3 \\
&+ 322095\, u_5\, u_6{}^2\, u_2{}^2\, u_4{}^4\, u_3 + 350\, u_8{}^3\, t^2\, u_2\, u_4{}^2\, u_3{}^2 - 250\, u_8{}^3\, u_2{}^3\, u_3{}^2\, u_4\, t \\
&- 1470130\, u_7{}^3\, u_3\, t\, u_4{}^3\, u_2{}^2 - 3247755\, u_5\, u_6{}^2\, u_3{}^3\, u_4{}^3\, u_2 + 2574630\, u_6\, u_7{}^2\, u_3{}^4\, u_4{}^2\, t \\
&- 1075760\, u_7{}^2\, u_8\, u_2{}^4\, u_3{}^2\, u_4 + 324720\, u_6{}^2\, u_8\, u_3{}^6\, u_2 - 1151435\, u_6\, u_7{}^2\, u_2{}^4\, u_4{}^3 \\
&- 9350\, u_6\, u_7{}^2\, u_4{}^5\, t^2 - 128400\, u_6\, u_7{}^2\, u_3{}^6\, u_2 + 1859340\, u_6{}^2\, u_8\, u_2{}^4\, u_4{}^3 -
\end{aligned}
$$

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\begin{aligned}
&- 744856\, u_6^3\, u_8\, u_2^5\, u_4 - 354116\, u_6^3\, u_8\, u_2^4\, u_3^2 - 591664\, u_6^3\, u_7\, u_2^3\, u_3^3 \\
&- 725440\, u_7^3\, u_6\, u_2^5\, u_3 + 219205\, u_5^2\, u_6^2\, u_2^3\, u_4^3 - 690920\, u_5^3\, u_8\, u_3^5\, u_2 \\
&+ 740750\, u_7^3\, u_5\, u_2^4\, u_3^2 - 1499205\, u_7^3\, u_5\, u_2^5\, u_4 + 3120390\, u_5^4\, u_2\, u_3^2\, u_4^3 \\
&- 368395\, u_7^4\, t\, u_2^4\, u_4 - 1299270\, u_5^3\, u_6\, u_3^5\, u_4 - 90\, u_8^3\, u_6\, u_2^5\, t \\
&+ 1859340\, u_6^4\, u_2^2\, u_4^3\, t + 1035412\, u_6^4\, u_2^3\, u_3^2\, u_4 - 287910\, u_6^3\, u_5\, u_3^5\, u_2 \\
&- 24246\, u_6^3\, u_7\, u_3^5\, t + 184510\, u_7^4\, t^2\, u_2^2\, u_4^2 + 368610\, u_7^4\, t\, u_2^3\, u_3^2 \\
&+ 11583\, u_6^4\, u_3^4\, t\, u_4 + 787540\, u_6^2\, u_7^2\, u_2^5\, u_4 + 184050\, u_6^2\, u_8^2\, u_3^4\, t^2 \\
&- 320\, u_6^2\, u_8^2\, u_4^3\, t^3 + 518266\, u_5^2\, u_7^2\, u_2^2\, u_3^4 + 743040\, u_5^2\, u_7^2\, u_4^4\, t^2 \\
&+ 744066\, u_5^2\, u_8^2\, u_2^4\, u_3^2 - 8289\, u_5^2\, u_8^2\, u_2^5\, u_4 + 1041800\, u_6^2\, u_7^2\, u_2^4\, u_3^2 \\
&- 367620\, u_6\, u_7^2\, u_8\, u_2^6 + 90\, u_7^2\, u_8^2\, u_2^5\, t - 4253985\, u_5^3\, u_7\, u_2^3\, u_4^3 \\
&- 540\, u_7\, u_8^2\, u_5\, u_2^6 + 4150095\, u_5^2\, u_7^2\, u_2^4\, u_4^2 - 1432080\, u_5^5\, u_3\, t\, u_4^3 \\
&- 367770\, u_6^5\, t^2\, u_2\, u_4^2 - 8289\, u_6^5\, u_3^2\, t^2\, u_4 + 371940\, u_5^2\, u_7^2\, u_6\, u_2^5 \\
&- 633570\, u_5^5\, u_2^2\, u_3\, u_4^2 - 1299270\, u_5^5\, u_2\, u_3^3\, u_4 - 378864\, u_6^5\, t\, u_2^2\, u_3^2 \\
&- 744856\, u_6^5\, t\, u_2^3\, u_4 + 150\, u_6^3\, u_8^2\, u_3^2\, t^3 + 58\, u_7^5\, t^3\, u_2\, u_3 - 818688\, u_5^3\, u_7^2\, u_2^4\, u_3 \\
&- 75\, u_6^3\, u_7^2\, u_4^2\, t^3 + 150\, u_8^3\, u_5^2\, t^2\, u_2^3 + 320\, u_6^3\, u_8^2\, t^2\, u_2^3 - 49248\, u_6^3\, u_5\, u_7\, u_2^5 \\
&- 373232\, u_6^3\, u_7^2\, u_2^4\, t - 378864\, u_5^2\, u_6^2\, u_8\, u_2^5 + 28053\, u_5^3\, u_8\, u_7\, u_2^5 \\
&+ 350945\, u_5^4\, u_8\, u_3^4\, t - 1115935\, u_5^2\, u_6^3\, u_4^3\, t^2 - 210580\, u_5^2\, u_6^3\, u_2^3\, u_3^2 \\
&- 149746\, u_5^3\, u_6^2\, u_2^2\, u_3^3 - 174\, u_7^4\, u_6\, u_3^2\, t^3 + 1035412\, u_5^2\, u_6^3\, u_2^4\, u_4 \\
&- 2525\, u_5^3\, u_8^2\, u_3^3\, t^2 - 733794\, u_7^3\, u_5^2\, u_3^3\, t^2 - 1089524\, u_5^4\, u_7\, u_2^2\, u_3^3 \\
&- 366770\, u_5^4\, u_8\, u_4^3\, t^2 - 1457721\, u_5^4\, u_8\, u_2^3\, u_3^2 + 11583\, u_5^4\, u_8\, u_2^4\, u_4 \\
&+ 225\, u_6^4\, u_8\, u_4^2\, t^3 + 5472\, u_6^4\, u_5\, u_2^4\, u_3 + 1061460\, u_5^6\, u_3^2\, t\, u_4 \\
&- 1457721\, u_5^2\, u_6^3\, u_3^4\, t + 245\, u_7^4\, u_6\, t^2\, u_2^3 + 368444\, u_6^4\, u_8\, u_2^4\, t \\
&+ 673430\, u_5^4\, u_6\, u_3^4\, u_2 + 1095060\, u_5^6\, t\, u_2\, u_4^2 + 729\, u_6^4\, u_7\, u_3^3\, t^2 \\
&- 690920\, u_5^5\, u_6\, u_3^3\, t - 287910\, u_5^5\, u_6\, u_2^3\, u_3 + 745040\, u_5^5\, u_7\, u_4^2\, t^2 \\
&- 180\, u_6^5\, u_7\, t^3\, u_3 - 90\, u_6^5\, u_8\, t^3\, u_2 - 3008880\, u_5\, u_6\, u_4^5\, u_3^3 + 744066\, u_6^4\, u_5^2\, u_3^2\, t^2 \\
&+ 184050\, u_5^4\, u_8^2\, t^2\, u_2^2 + 1107990\, u_5^4\, u_7^2\, u_3^2\, t^2 + 770\, u_5^3\, u_7^3\, t^2\, u_2^2 \\
&- 354116\, u_6^4\, u_5^2\, u_2^3\, t + 2051110\, u_5^4\, u_6^2\, u_4^2\, t^2 + 383550\, u_5^4\, u_7^2\, u_2^3\, t \\
&- 100\, u_5^4\, u_8^2\, t^3\, u_4 + 310\, u_7^4\, u_5^2\, t^3\, u_2 + 550\, u_5^3\, u_7^3\, t^3\, u_4 + 120\, u_6^4\, u_7^2\, t^3\, u_2 \\
&+ 746840\, u_5^5\, u_6^2\, t^2\, u_3 + 324720\, u_5^6\, u_6\, t\, u_2^2 - 1119060\, u_5^6\, u_6\, t^2\, u_4 \\
&- 744440\, u_5^6\, u_7\, t^2\, u_3 - 363420\, u_5^6\, u_8\, t^2\, u_2 - 1100\, u_5^4\, u_7^2\, u_6\, t^3 + 400\, u_5^5\, u_7\, u_8\, t^3 \\
&+ 1000\, u_5^3\, u_6^3\, u_7\, t^3 - 370820\, u_5^4\, u_6^3\, t^2\, u_2 - 13\, u_7^4\, u_8\, t^3\, u_2^2 - 13\, u_8^3\, u_6^2\, t^3\, u_2^2 \\
&+ 1844535\, u_5^4\, u_6\, u_2^3\, u_4^2 - 707640\, u_5^7\, t\, u_2\, u_3 + 9627175\, u_5^2\, u_7\, u_2^2\, u_4^4\, u_3 \\
&+ 2758860\, u_5^2\, u_6\, u_4^4\, u_3^2\, u_2 - 1536725\, u_5\, u_6\, u_8\, u_2^3\, u_4^3\, u_3 \\
&+ 1471780\, u_5\, u_6\, u_8\, u_3^5\, u_2\, u_4 - 1383880\, u_5\, u_7^2\, u_3^5\, u_2\, u_4 - 1171515\, u_5^2\, u_7\, u_3^3\, u_4^3\, u_2 \\
&+ 683020\, u_6^2\, u_8\, u_3^2\, t\, u_4^3\, u_2 + 3761270\, u_6^2\, u_7\, u_3\, t\, u_4^4\, u_2 - 236520\, u_6^2\, u_8\, u_2^2\, u_3^4\, u_4 \\
&- 2997465\, u_6^2\, u_7\, u_3^3\, u_4^3\, t + 2861960\, u_6\, u_7\, u_8\, u_2^3\, u_3^3\, u_4 \\
&+ 5186945\, u_5\, u_6\, u_7\, u_3^2\, u_4^4\, t - 2244420\, u_5\, u_8^2\, u_2^3\, u_3^3\, u_4 + 725240\, u_5\, u_7\, u_8\, u_3^6\, u_2 \\
&+ 807480\, u_5\, u_6\, u_7\, u_3^6\, u_4 - 1471755\, u_5\, u_7\, u_8\, u_2^4\, u_4^3 - 736140\, u_5\, u_7\, u_8\, u_4^5\, t^2 \\
&- 699840\, u_6\, u_7\, u_8\, u_2^2\, u_3^5 + 4302815\, u_5\, u_6\, u_7\, u_2^3\, u_4^4 + 739200\, u_6\, u_7\, u_8\, u_4^4\, t^2\, u_3 \\
&- 6876865\, u_5\, u_6\, u_7\, u_2^2\, u_4^3\, u_3^2 - 1539655\, u_5\, u_6\, u_7\, u_4^5\, t\, u_2 \\
&- 5201180\, u_5\, u_7\, u_8\, u_2^2\, u_3^4\, u_4 - 2195695\, u_5\, u_8^2\, u_3^3\, t\, u_4^2\, u_2 \\
&- 5455\, u_5\, u_8^2\, u_3\, t\, u_4^3\, u_2^2 - 740340\, u_5\, u_8^2\, u_4^4\, t^2\, u_3 - 67815\, u_5\, u_7\, u_8\, u_3^2\, t\, u_4^3\, u_2 \\
&- 8955\, u_7\, u_8^2\, u_2^3\, u_4^2\, u_3\, t - 100\, u_7\, u_8^2\, u_3^3\, t^2\, u_4^2 + 11150\, u_7\, u_8^2\, u_3^3\, t\, u_4\, u_2^2 \\
&- 716365\, u_7^2\, u_8\, u_2^3\, u_4^3\, t - 1451685\, u_6^2\, u_8\, u_2^2\, u_4^4\, t - 3045545\, u_6^2\, u_8\, u_2^3\, u_4^2\, u_3^2 \\
&+ 1455765\, u_7^2\, u_8\, u_3^2\, t\, u_4^2\, u_2^2 - 1479380\, u_7^2\, u_8\, u_3^4\, t\, u_4\, u_2 \\
&- 368640\, u_7^2\, u_8\, u_3^2\, t^2\, u_4^3 - 1462540\, u_6\, u_8^2\, u_3^2\, t\, u_4^2\, u_2^2 \\
&+ 1456930\, u_6\, u_8^2\, u_3^4\, t\, u_4\, u_2 - 32700\, u_6\, u_7\, u_8\, u_3^5\, t\, u_4 - 2173670\, u_6\, u_7\, u_8\, u_2^4\, u_4^2\, u_3 \\
&+ 763170\, u_5\, u_8^2\, u_2^4\, u_4^2\, u_3 - 2700\, u_7\, u_8^2\, u_2^5\, u_4\, u_3 - 2800\, u_7\, u_8^2\, u_3^5\, t\, u_2 +
\end{aligned}
$$

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\begin{aligned}
&+ 340\, u_7\, u_8{}^2\, t^2\, u_2\, u_4{}^3\, u_3 + 99795\, u_6\, u_7\, u_8\, u_3{}^3\, t\, u_4{}^2\, u_2 + 1413765\, u_6\, u_7\, u_8\, u_3\, t\, u_4{}^3\, u_2{}^2 \\
&- 367254\, u_7{}^4\, u_3{}^2\, t^2\, u_4\, u_2 - 739962\, u_6{}^4\, u_3{}^2\, t\, u_4{}^2\, u_2 + 7965345\, u_5\, u_6{}^2\, u_7\, u_3{}^2\, t\, u_4{}^2\, u_2 \\
&+ 175\, u_8{}^3\, u_5\, t^2\, u_2{}^2\, u_4\, u_3 + 36330\, u_6{}^3\, u_7\, u_3{}^3\, t\, u_4\, u_2 - 1536725\, u_6{}^3\, u_5\, u_3\, t\, u_4{}^3\, u_2 \\
&- 680\, u_7{}^3\, u_8\, t\, u_2{}^4\, u_3 + 4502365\, u_5{}^2\, u_6{}^2\, t\, u_2\, u_4{}^4 + 6826864\, u_5{}^2\, u_6{}^2\, u_2{}^2\, u_3{}^2\, u_4{}^2 \\
&+ 1402704\, u_5{}^3\, u_8\, u_2{}^3\, u_4{}^2\, u_3 + 4326150\, u_5{}^3\, u_8\, u_2{}^2\, u_3{}^3\, u_4 - 196\, u_8{}^3\, u_6\, t^2\, u_2{}^3\, u_4 \\
&- 56\, u_8{}^3\, u_6\, t^3\, u_2\, u_4{}^2 + 94\, u_8{}^3\, u_6\, t^2\, u_2{}^2\, u_3{}^2 + 738420\, u_6{}^3\, u_8\, t^2\, u_2\, u_4{}^3 \\
&+ 747772\, u_6{}^3\, u_8\, t\, u_2{}^2\, u_3{}^2\, u_4 - 3735664\, u_6{}^3\, u_7\, t\, u_2{}^2\, u_4{}^2\, u_3 - 366789\, u_6{}^3\, u_8\, u_3{}^2\, t^2\, u_4{}^2 \\
&+ 4380240\, u_5{}^3\, u_6\, u_3\, t\, u_4{}^4 + 1402704\, u_6{}^3\, u_5\, u_3{}^3\, t\, u_4{}^2 + 1503169\, u_5\, u_6{}^2\, u_7\, u_2{}^3\, u_3{}^2\, u_4 \\
&- 2294185\, u_5{}^3\, u_6\, u_2\, u_3{}^3\, u_4{}^2 - 3247755\, u_5{}^3\, u_6\, u_2{}^2\, u_4{}^3\, u_3 + 240\, u_8{}^3\, u_5\, t\, u_2{}^4\, u_3 \\
&- 4620\, u_6{}^3\, u_8\, u_2{}^3\, u_4{}^2\, t - 1693716\, u_6{}^3\, u_5\, u_2{}^3\, u_4{}^2\, u_3 - 2190727\, u_6{}^3\, u_5\, u_2{}^2\, u_3{}^3\, u_4 \\
&+ 4630\, u_7{}^3\, u_6\, t\, u_2{}^3\, u_3\, u_4 + 588\, u_7{}^3\, u_8\, t^2\, u_2\, u_3{}^3 - 737200\, u_7{}^3\, u_5\, t^2\, u_2\, u_4{}^3 \\
&- 2194442\, u_7{}^3\, u_5\, t\, u_2{}^2\, u_3{}^2\, u_4 - 731898\, u_7{}^3\, u_6\, u_3{}^3\, t^2\, u_4 + 2203895\, u_5{}^3\, u_8\, u_3{}^3\, t\, u_4{}^2 \\
&+ 729396\, u_7{}^3\, u_5\, u_3{}^2\, t^2\, u_4{}^2 - 1606824\, u_6{}^3\, u_7\, u_2{}^4\, u_3\, u_4 - 5007155\, u_5{}^2\, u_6{}^2\, u_3{}^2\, t\, u_4{}^3 \\
&- 738065\, u_6{}^2\, u_7\, u_8\, t^2\, u_2\, u_4{}^2\, u_3 + 736197\, u_6\, u_7{}^2\, u_8\, u_3{}^2\, t^2\, u_4\, u_2 \\
&+ 1062\, u_6\, u_8{}^2\, u_7\, t^2\, u_2{}^2\, u_4\, u_3 + 4382495\, u_5{}^2\, u_8\, u_7\, u_3{}^3\, t\, u_4\, u_2 \\
&- 715580\, u_5{}^2\, u_6\, u_7\, u_3{}^5\, u_2 - 8762910\, u_5{}^2\, u_6\, u_7\, u_3\, t\, u_4{}^3\, u_2 - 739962\, u_5{}^2\, u_8\, u_6\, u_2{}^4\, u_4{}^2 \\
&+ 774854\, u_5{}^2\, u_8\, u_6\, u_3{}^2\, t\, u_4{}^2\, u_2 - 736400\, u_5{}^2\, u_8\, u_7\, u_3{}^5\, t - 711024\, u_5\, u_7{}^2\, u_6\, u_3{}^5\, t \\
&- 1540607\, u_5\, u_6{}^2\, u_8\, u_3{}^3\, t\, u_4\, u_2 - 3048548\, u_5\, u_7{}^2\, u_6\, u_3{}^3\, t\, u_4\, u_2 \\
&- 728044\, u_5\, u_7{}^2\, u_8\, t^2\, u_2\, u_4{}^2\, u_3 + 1462770\, u_5\, u_6{}^2\, u_8\, u_3{}^5\, t - 368496\, u_6\, u_7{}^2\, u_8\, u_3{}^4\, t^2 \\
&+ 50\, u_6\, u_7{}^2\, u_8\, u_4{}^3\, t^3 + 1525428\, u_5{}^2\, u_8\, u_7\, u_2{}^3\, u_3{}^3 + 1101310\, u_5{}^2\, u_8\, u_6\, u_4{}^4\, t^2 \\
&- 1156800\, u_5{}^2\, u_8\, u_6\, u_2{}^2\, u_3{}^4 - 2089890\, u_5\, u_7{}^2\, u_6\, u_2{}^3\, u_3{}^3 + 98502\, u_5\, u_6{}^2\, u_7\, u_3{}^4\, t\, u_4 \\
&- 367113\, u_6{}^2\, u_8{}^2\, u_3{}^2\, t^2\, u_4\, u_2 + 379758\, u_6{}^2\, u_7{}^2\, t\, u_2\, u_3{}^4 - 364760\, u_5{}^2\, u_8{}^2\, t\, u_2\, u_3{}^4 \\
&+ 20028\, u_5\, u_6\, u_8\, u_7\, t\, u_2\, u_3{}^4 + 733632\, u_5\, u_6\, u_8{}^2\, t^2\, u_2\, u_4{}^2\, u_3 \\
&- 1374267\, u_5{}^2\, u_7{}^2\, u_3{}^2\, t\, u_4{}^2\, u_2 + 5253426\, u_5\, u_7{}^2\, u_6\, t\, u_2{}^2\, u_4{}^2\, u_3 \\
&+ 742474\, u_5\, u_7{}^2\, u_8\, t\, u_2{}^2\, u_3{}^3 + 721840\, u_6{}^2\, u_7\, u_8\, t\, u_2{}^2\, u_3{}^3 \\
&+ 2822307\, u_5{}^2\, u_6\, u_7\, u_2{}^3\, u_4{}^2\, u_3 - 2530\, u_7\, u_8{}^2\, u_5\, u_3{}^2\, t^2\, u_4\, u_2 \\
&- 1465300\, u_5\, u_7{}^2\, u_6\, u_3\, t^2\, u_4{}^3 + 734400\, u_5\, u_7{}^2\, u_8\, u_3{}^3\, t^2\, u_4 + 440\, u_6\, u_8{}^2\, u_7\, t\, u_2{}^4\, u_3 \\
&+ 2269532\, u_5\, u_6{}^2\, u_8\, t\, u_2{}^2\, u_4{}^2\, u_3 + 236\, u_6\, u_8{}^2\, u_7\, u_3\, t^3\, u_4{}^2 + 734360\, u_6\, u_7{}^2\, u_8\, t\, u_2{}^4\, u_4 \\
&- 733120\, u_6\, u_7{}^2\, u_8\, t\, u_2{}^3\, u_3{}^2 - 3562284\, u_5{}^2\, u_8\, u_7\, t\, u_2{}^2\, u_4{}^2\, u_3 \\
&+ 2206820\, u_5{}^2\, u_8\, u_7\, u_3\, t^2\, u_4{}^3 - 4002085\, u_5{}^2\, u_7{}^2\, u_2{}^2\, u_4{}^3\, t + 4179\, u_5\, u_7{}^2\, u_8\, t\, u_2{}^3\, u_3\, u_4 \\
&- 3632765\, u_5{}^2\, u_8\, u_6\, u_3{}^4\, t\, u_4 + 736308\, u_6{}^2\, u_7\, u_8\, u_3{}^3\, t^2\, u_4 \\
&- 2206122\, u_5\, u_6\, u_8\, u_7\, u_3{}^2\, t^2\, u_4{}^2 - 9080\, u_5\, u_6\, u_8\, u_7\, t^2\, u_2\, u_4{}^3 \\
&- 1495356\, u_5\, u_6\, u_8\, u_7\, u_2{}^3\, u_4{}^2\, t - 1470819\, u_5{}^2\, u_7{}^2\, u_2{}^3\, u_3{}^2\, u_4 \\
&+ 2906650\, u_5{}^3\, u_7\, u_2\, u_3{}^4\, u_4 + 2213245\, u_5{}^3\, u_7\, t\, u_2\, u_4{}^4 - 4144849\, u_5{}^3\, u_7\, u_2{}^2\, u_3{}^2\, u_4{}^2 \\
&- 366789\, u_5{}^2\, u_8{}^2\, u_2{}^3\, u_4{}^2\, t - 4102\, u_7\, u_8{}^2\, u_5\, t\, u_2{}^3\, u_3{}^2 + 2943\, u_7\, u_8{}^2\, u_5\, t\, u_2{}^4\, u_4 \\
&+ 9300\, u_6{}^2\, u_7\, u_8\, t\, u_2{}^3\, u_3\, u_4 + 2235376\, u_5\, u_6{}^2\, u_8\, u_2{}^4\, u_3\, u_4 - 458\, u_6\, u_8{}^2\, u_7\, t^2\, u_2\, u_3{}^3 \\
&- 3701459\, u_5{}^2\, u_6\, u_7\, u_3{}^3\, t\, u_4{}^2 - 94\, u_7{}^2\, u_8{}^2\, t^2\, u_2{}^2\, u_3{}^2 - 10\, u_7{}^2\, u_8{}^2\, u_3{}^2\, t^3\, u_4 \\
&+ 196\, u_7{}^2\, u_8{}^2\, t^2\, u_2{}^3\, u_4 + 183259\, u_6{}^2\, u_8{}^2\, t^2\, u_2{}^2\, u_4{}^2 - 356810\, u_6{}^2\, u_7{}^2\, t^2\, u_2\, u_4{}^3 \\
&+ 1850668\, u_6{}^2\, u_7{}^2\, t\, u_2{}^2\, u_3{}^2\, u_4 + 742840\, u_5{}^2\, u_8{}^2\, u_3{}^2\, t^2\, u_4{}^2 \\
&+ 2567280\, u_5{}^2\, u_8{}^2\, t\, u_2{}^2\, u_3{}^2\, u_4 + 301\, u_5\, u_6\, u_8{}^2\, t\, u_2{}^3\, u_3\, u_4 \\
&- 738210\, u_5\, u_6\, u_8{}^2\, u_3{}^3\, t^2\, u_4 + 2512640\, u_5{}^2\, u_7{}^2\, u_3{}^4\, t\, u_4 \\
&- 2990428\, u_5\, u_6\, u_8\, u_7\, t\, u_2{}^2\, u_3{}^2\, u_4 - 377090\, u_6{}^2\, u_7{}^2\, u_2{}^3\, u_4{}^2\, t \\
&- 739278\, u_5\, u_6{}^2\, u_7\, u_8\, t^2\, u_2\, u_3{}^2 - 773835\, u_5\, u_6{}^2\, u_7{}^2\, t\, u_2{}^3\, u_3 \\
&+ 3324101\, u_5{}^2\, u_7{}^2\, u_6\, t\, u_2{}^2\, u_3{}^2 - 4374060\, u_5{}^2\, u_7{}^2\, u_6\, t\, u_2{}^3\, u_4 + 10\, u_8{}^3\, u_5{}^2\, t^3\, u_2\, u_4 \\
&- 196\, u_6{}^3\, u_8{}^2\, t^3\, u_2\, u_4 + 127\, u_5\, u_7{}^2\, u_6\, u_8\, u_3\, t^3\, u_4 + 143\, u_6\, u_8{}^2\, u_7\, u_5\, t^3\, u_2\, u_4 \\
&+ 2577970\, u_5{}^2\, u_7{}^2\, u_6\, t^2\, u_2\, u_4{}^2 - 367113\, u_5{}^2\, u_8{}^2\, u_6\, t^2\, u_2{}^2\, u_4 \\
&+ 2973971\, u_5{}^2\, u_8\, u_6\, u_7\, t\, u_2{}^3\, u_3 + 3272\, u_5\, u_7{}^2\, u_6\, u_8\, t^2\, u_2{}^2\, u_3 + 9\, u_6{}^2\, u_7\, u_8{}^2\, t^3\, u_2\, u_3 \\
&- 60\, u_5{}^2\, u_8{}^2\, u_7\, u_3\, t^3\, u_4 + 369715\, u_6{}^3\, u_7{}^2\, t^2\, u_2{}^2\, u_4 - 375189\, u_5{}^2\, u_7{}^2\, u_8\, u_2{}^4\, t -
\end{aligned}
$$

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\begin{aligned}
&- 2208929\, u_5\, {u_6}^2\, {u_7}^2\, t^2\, u_2\, u_4\, u_3 - 755926\, {u_5}^2\, {u_6}^2\, u_7\, t\, u_2\, {u_3}^3\\
&- 429928\, {u_5}^2\, {u_6}^2\, u_7\, {u_2}^4\, u_3 + 24\, {u_6}^2\, {u_7}^2\, u_8\, {u_3}^2\, t^3 - 729916\, u_5\, {u_6}^2\, u_7\, u_8\, {u_2}^4\, t\\
&- 24620\, {u_6}^3\, u_5\, u_7\, t^2\, u_2\, {u_4}^2 - 728690\, {u_5}^3\, u_8\, u_7\, t^2\, u_2\, {u_4}^2 - 1475585\, {u_5}^3\, u_8\, u_7\, {u_3}^2\, t^2\, u_4\\
&- 737800\, {u_7}^3\, u_5\, u_6\, t^2\, {u_2}^2\, u_4 - 2195695\, {u_5}^3\, u_8\, u_6\, u_3\, t^2\, {u_4}^2\\
&- 1462540\, {u_5}^2\, {u_6}^2\, u_8\, t^2\, u_2\, {u_4}^2 - 415\, {u_6}^2\, {u_7}^2\, u_8\, t^2\, {u_2}^3\\
&- 4470273\, {u_5}^2\, {u_6}^2\, u_7\, t\, {u_2}^2\, u_3\, u_4 + 3542894\, {u_5}^3\, {u_7}^2\, t\, {u_2}^2\, u_3\, u_4\\
&+ 7249510\, {u_5}^3\, u_6\, u_7\, t\, {u_2}^2\, {u_4}^2 + 770\, {u_6}^3\, u_7\, u_8\, t^2\, {u_2}^2\, u_3 - 1496884\, {u_6}^3\, u_5\, u_8\, t\, {u_2}^3\, u_3\\
&- 1456521\, {u_6}^3\, u_5\, u_7\, t\, {u_2}^2\, {u_3}^2 + 735042\, {u_7}^3\, u_5\, u_6\, t^2\, u_2\, {u_3}^2\\
&- 2228049\, {u_5}^3\, u_8\, u_7\, t\, {u_2}^2\, {u_3}^2 + 1451301\, {u_5}^3\, u_8\, u_7\, t\, {u_2}^3\, u_4\\
&+ 765258\, {u_5}^2\, {u_6}^2\, u_8\, t\, {u_2}^2\, {u_3}^2 + 747772\, {u_5}^2\, {u_6}^2\, u_8\, t\, {u_2}^3\, u_4 + 789\, {u_7}^3\, {u_6}^2\, u_3\, t^3\, u_4\\
&- 182\, {u_7}^4\, u_5\, u_3\, t^3\, u_4 - 145\, {u_7}^4\, u_6\, t^3\, u_2\, u_4 - 2853470\, {u_5}^4\, u_7\, {u_3}^3\, t\, u_4\\
&+ 1449999\, {u_5}^3\, u_8\, u_6\, {u_2}^4\, u_3 - 734229\, {u_5}^3\, {u_8}^2\, t\, {u_2}^3\, u_3 - 2190727\, {u_5}^3\, {u_6}^2\, {u_2}^3\, u_3\, u_4\\
&- 3045545\, {u_5}^2\, {u_6}^3\, t\, {u_2}^2\, {u_4}^2 - 4439365\, {u_5}^4\, u_6\, t\, u_2\, {u_4}^3 + 335615\, {u_5}^4\, u_6\, {u_2}^2\, {u_3}^2\, u_4\\
&+ 366993\, {u_6}^4\, u_8\, t^2\, u_2\, {u_3}^2 + 757408\, {u_6}^4\, u_7\, t\, {u_2}^3\, u_3 - 735886\, {u_7}^3\, {u_5}^2\, t\, {u_2}^3\, u_3\\
&- 1657\, {u_7}^3\, {u_6}^2\, t^2\, {u_2}^2\, u_3 + 2984832\, {u_5}^3\, u_6\, u_7\, {u_3}^2\, t\, u_4\, u_2 + 301\, {u_6}^3\, u_5\, u_8\, t^2\, u_2\, u_4\, u_3\\
&+ 743125\, {u_7}^3\, u_5\, u_6\, {u_2}^4\, t - 121\, {u_7}^3\, u_5\, u_8\, t^2\, {u_2}^3 + 1449999\, {u_6}^4\, u_5\, t\, u_2\, {u_3}^3\\
&- 3500427\, {u_5}^2\, {u_6}^3\, {u_3}^2\, t\, u_4\, u_2 - 738210\, {u_5}^3\, {u_8}^2\, t^2\, u_2\, u_4\, u_3\\
&+ 737616\, {u_7}^3\, {u_5}^2\, t^2\, u_2\, u_4\, u_3 + 731240\, {u_5}^5\, u_8\, u_3\, t^2\, u_4 - 1473678\, {u_5}^3\, {u_7}^2\, u_6\, t^2\, u_2\, u_3\\
&+ 220\, u_5\, {u_6}^3\, u_7\, u_8\, t^3\, u_2 - 130\, {u_5}^3\, {u_8}^2\, u_6\, t^3\, u_3 + 94\, {u_5}^2\, {u_8}^2\, {u_6}^2\, t^3\, u_2\\
&- 2827126\, {u_5}^4\, u_6\, u_7\, t\, {u_2}^2\, u_3 - 2244420\, {u_5}^3\, {u_6}^3\, u_3\, t^2\, u_4 - 1486476\, {u_5}^3\, {u_6}^2\, u_7\, {u_3}^2\, t^2\\
&+ 368862\, {u_5}^2\, {u_6}^3\, u_8\, t^2\, {u_2}^2 + 1456930\, {u_5}^4\, u_8\, u_6\, t^2\, u_2\, u_4 + 2209452\, {u_5}^2\, {u_6}^3\, u_7\, t^2\, u_2\, u_3\\
&- 738820\, {u_6}^4\, u_5\, u_7\, t^2\, {u_2}^2 - 236520\, {u_5}^4\, {u_6}^2\, t\, {u_2}^2\, u_4 - 1156800\, {u_5}^4\, {u_6}^2\, t\, u_2\, {u_3}^2\\
&- 727344\, {u_5}^4\, u_8\, u_6\, {u_2}^3\, t - 250\, {u_5}^2\, {u_6}^3\, u_8\, t^3\, u_4 - 40\, {u_5}^3\, {u_8}^2\, u_7\, t^3\, u_2\\
&- 1500\, {u_6}^4\, u_5\, u_7\, t^3\, u_4 + 240\, {u_6}^4\, u_5\, u_8\, t^3\, u_3 - 1723605\, {u_5}^3\, u_6\, u_7\, {u_2}^4\, u_4\\
&- 1540607\, {u_5}^3\, u_8\, u_6\, t\, {u_2}^2\, u_3\, u_4.
\end{aligned}
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